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TECHNICAL ARTICLE

Quantum Sociodynamics for AI-Supported Sustainable City Transitions: A Gauge-Covariant Method for Modelling Symbolic Mediation and Public Trust

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ABSTRACT

Quantum Sociodynamics (QSD) is introduced as a computational method for modelling symbolic influence and trust in artificial-intelligence-supported sustainable city transitions. Its central object, the Symbolic Mediation Field, represents how actor orientations, source–audience relations and communication histories shape signal effects. Actor states are encoded by complex variables ψ_i, symbolic transport by U(1) link variables U_{ij}, signed coupling by q_i and external interventions by J_i. Gauge-invariant measures capture local alignment, path transport and loop holonomy. Formal analysis establishes invariance under local re-description, path independence under zero holonomy and a classical DeGroot–Friedkin–Johnsen limit. Deterministic simulations demonstrate response reversal, order effects and persistent relational change after institutional shocks. A 27-record heat-pump corpus illustrates application to AI-assisted urban decarbonisation governance and identifies validation requirements.

KEYWORDS Artificial Intelligence (AI), Sustainable Building Systems

1. Introduction

1.1. The unresolved problem in network models of influence

Artificial intelligence is becoming embedded in energy management, urban planning, environmental monitoring, mobility, digital twins and public-service decision systems. Its contribution to sustainable development can be substantial, but its effects are not automatically beneficial: transparency, accountability, distributive consequences and institutional legitimacy shape whether technically capable systems are accepted and used responsibly (Allam and Dhunny 2019; Son *et al.* 2023; Vinuesa *et al.* 2020). Recent work in the editor-suggested journal set also demonstrates the expanding use of AI, digital twins and human-centred predictive modelling in sustainable built environments, while stressing interpretability, occupant feedback and stakeholder trust (Abdalla et al., 2026; Aziz et al., 2025; Razak et al., 2025; Samaei and Riffat, 2026).

Urban sustainability therefore presents a modelling problem that is simultaneously technical and relational. The same heat-transition forecast, smart-grid recommendation or automated policy message can reassure one community, fail with another and intensify distrust in a third. Model accuracy alone cannot explain this variation because responses depend on source identity, prior institutional experience, communication route, perceived fairness and the accumulated history of earlier interventions.

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Existing approaches address parts of the problem. Artificial-intelligence and digital-twin research provides prediction, optimisation and scenario simulation for urban systems, while network models describe influence, diffusion and trust learning (Bibri *et al.* 2024; Tzachor *et al.* 2022). Yet the symbolic transformation of a signal during source-to-audience transport is commonly treated as an external covariate, a message feature or a static trust weight rather than as an endogenous relational state.

This Technical Article develops that missing computational object, derives its invariant properties, tests its mechanisms in deterministic simulations and demonstrates its use in an urban heat-decarbonisation controversy. The method is designed as a feature-engineering, simulation and interpretability layer for artificial-intelligence-supported sustainable-city decision systems.

1.2. The Symbolic Mediation Field

Quantum Sociodynamics (QSD) is organised around the **Symbolic Mediation Field** (SMF):

> *A Symbolic Mediation Field is a dynamically evolving relational structure through which communicative signals acquire differential capacities to alter belief, affect, institutional trust and behavioural orientation. Its state is produced by accumulated interaction among actors, sources, institutions, material events and counter-messages. The effect of a signal depends jointly on the current field, the actor's signed coupling, the source relation, the route of transmission and the ordered history of earlier interactions.*

The SMF is not a synonym for context. Context is often represented as a set of background covariates. The SMF is endogenous: communication and institutional events change the field that conditions later communication. Nor is the field simply a frame. A frame is an organisation of meaning within a message or discourse; in QSD, a frame is a symbolic excitation entering a larger relational medium. The explanatory target is therefore not only message content or individual response, but the emergence of meso- and macro-level credibility structures: aligned communities, fragmented trust topologies, path-dependent institutional reputations and persistent inconsistency around communication cycles.

1.3. Why a gauge architecture?

QSD draws structurally from the gauge architecture of quantum electrodynamics (QED), not from generic claims that society is uncertain, complex or "quantum". The relevant inheritance is a rigorous interaction grammar: locally represented states, mediator variables on relations, coupling strengths, covariant transport and observables that remain unchanged under arbitrary local re-description.

This choice addresses a genuine modelling problem. Different communities may code the same interpretive orientation from different baselines; a formal result should not depend on those arbitrary local choices. What is sociologically meaningful is relational alignment after transport, accumulated transformation along a route and inconsistency around a closed communication cycle. Gauge invariance separates such substantive quantities from representational conventions.

QSD does not claim that people are particles, that messages are photons or that social systems literally obey quantum electrodynamics. It proposes a QED-derived, U(1)-type network method whose social correspondences are explicit and whose predictions can be compared with simpler models. In sustainable-city applications, the method can represent residents, institutions, media, utilities and AI decision systems as interacting nodes while preserving the distinction between technical recommendation, source relation and communication history.

1.4. Contribution and priority claim

The article makes five linked contributions.

1. **A new explanatory object.** It defines symbolic mediation as a state-bearing and coevolving relational field rather than a message attribute or static context.

2. **A new network representation.** It replaces fixed scalar influence weights with meaning-bearing link variables that transform actor states during transmission.
3. **New gauge-invariant observables.** It introduces relational alignment, path transport and closed-loop holonomy as measures of credibility compatibility, route dependence and unresolved cycle inconsistency.
4. **A nesting result.** It shows that standard weighted influence dynamics arise as a flat, real-valued, static-link limit of QSD.
5. **A falsifiable empirical programme.** It derives conditions for response reversal, ordered exposure, field memory and route curvature and states when the additional structure should be rejected.

A targeted audit of framing theory, Science and Technology Studies, misinformation psychology, opinion dynamics, sociophysics, quantum cognition, quantum social ontology, quantum-field-inspired decision models, signed-network gauge models and AI-enabled urban modelling found no earlier framework combining a state-bearing symbolic mediator field, signed actor coupling, local gauge covariance, dynamic source–audience transport, path and loop observables, field feedback and a classical influence limit in one computational method. QSD is therefore advanced as a distinct technical contribution rather than a relabelling of existing approaches.

2. Technical Background and Methodological Gap

2.1. AI-supported urban transitions and public trust

Artificial-intelligence-supported urban systems increasingly integrate sensor data, optimisation, forecasting and digital-twin simulation. Reviews of smart-city and urban-planning applications emphasise that wider adoption requires collaboration among government, technical experts and affected communities, while digital-twin research identifies governance, uncertainty and social representation as continuing limitations (Bibri *et al.* 2024; Son *et al.* 2023; Tzachor *et al.* 2022). Related applications in climate-resilient infrastructure also demonstrate the use of AI-guided optimisation and digital-twin methods to assess and improve the performance of low-carbon coastal systems (Riffat *et al.* 2025). Related studies report AIoT-enabled building control, digital-twin integration and AI-guided optimisation for low-carbon infrastructure, reinforcing the need to connect technical performance with user-centred governance and deployment conditions (Onweh *et al.* 2025; Riffat *et al.* 2025).

The technical gap is not the absence of predictive models; it is the absence of a formal layer for representing how recommendations acquire different meanings as they move through institutions, platforms and communities. Conventional AI pipelines may model the probability of acceptance, but they seldom separate exposure magnitude, actor coupling, source-conditioned transformation and route inconsistency as distinct state variables.

QSD addresses this gap by treating symbolic relations as time-varying transporters that can be estimated from longitudinal exposure, source-trust, media and behavioural data. Gauge-invariant features can then be supplied to machine-learning, agent-based, digital-twin or decision-support systems without allowing predictions to depend on arbitrary local coding baselines.

2.2. Network influence, trust and message interaction

DeGroot and Friedkin–Johnsen models treat opinion change as repeated weighted averaging, while network-autocorrelation and dynamic-network models represent relational dependence and changing ties (Butts 2025; Degroot 1974; Friedkin and Johnsen 1990; Leenders 2002). Formal trust models increasingly treat trust as learned and socially interdependent rather than as a fixed personality attribute (Meylahn *et al.* 2024).

Message-interaction models also show that communication cannot always be treated as independent input (Mai *et al.* 2026). QSD adds a different structure: the relation itself carries a

transformation, the recipient has a signed issue-specific coupling and institutional events can update the relational medium even when network topology and message content remain unchanged.

For AI-supported urban governance, this shift redirects analysis from isolated prediction to system-level interpretability. Persistent distrust, fragmented credibility and polarised response can emerge from the coevolution of actor states and communication relations, providing information that a high-accuracy classifier or optimiser may otherwise miss.

2.3. Signed networks, structural balance and cycle consistency

Signed-network models distinguish positive and negative relations and use cycle structure to analyse balance. Gauge transformations can identify equivalent sign configurations, while frustrated cycles indicate incompatibility that cannot be removed by relabelling (Facchetti *et al.* 2011). QSD generalises this insight from binary signs to continuous symbolic transformations. A link phase can represent more than attraction or hostility: it can encode source-conditioned reinterpretation, credibility displacement and route-specific semantic shift.

The corresponding loop quantity is holonomy. In a flat field, transport between two nodes is independent of route. In a curved field, different routes accumulate different transformations. This supplies a compact mathematical representation of a common sociological phenomenon: two actors can receive the "same" proposition yet receive non-equivalent social objects because the proposition has travelled through different institutional and interpersonal chains.

2.4. Quantum-like social models and the exact boundary

Quantum cognition uses Hilbert-space probability to represent contextuality and question-order effects (Busemeyer and Bruza 2012; Wang *et al.* 2014). Wendt (2015) proposes a quantum social ontology; Bagarello, Basieva and Khrennikov (2018) model belief stabilisation through interaction with an informational environment; Khrennikov (2020) develops quantum-field and information analogies for social processes. These works establish legitimate precedents for quantum-derived mathematics in social inquiry.

QSD is neither a restatement of those models nor a claim about macroscopic physical quantum states. Its unit of formalisation is the **source–actor communication relation on a coevolving social network**. The distinctive architecture combines actor states, signed coupling, dynamic link transport, local gauge freedom, route observables, loop curvature and feedback. Quantum probability is optional: it should be introduced only if non-classical constraints outperform classical alternatives. The formal novelty of QSD rests on the gauge-covariant network architecture developed below.

2.5. Novelty matrix

Table 1 compares QSD with adjacent computational approaches and identifies the specific modelling gap addressed by the proposed method.

Table 1. Comparison of QSD with Adjacent Computational Approaches.

Approach	Primary strength	Structural limitation addressed by QSD
DeGroot / Friedkin–Johnsen influence	Weighted updating and consensus with predisposition	Links are generally scalar rather than meaning-bearing transporters
Network autocorrelation	Relational dependence and model-based weight matrices	Route transformation and cycle inconsistency are not explicit observables
Dynamic-network influence	Coevolution of ties and attributes	A topologically stable relation can change symbolically without appearing or disappearing
Formal trust models	Learning whether an institution is trustworthy	Trust-conditioned signal transformation is not represented as gauge transport

Table 1. (continued)

Approach	Primary strength	Structural limitation addressed by QSD
Message-interaction models	Non-independent effects of multiple messages	Interaction history is not necessarily stored in a coevolving relational field
Signed-network balance	Positive/negative ties and frustrated cycles	Binary signs are generalised to continuous symbolic phase and route holonomy
Quantum cognition	Contextual probability and ordered judgments	Focus is primarily individual cognition rather than networked symbolic mediation
Quantum Sociodynamics	**Symbolic influence in coevolving social networks**	**Unifies state, signed coupling, dynamic transport, local gauge freedom, route curvature and feedback**
AI and urban digital-twin models	Prediction, optimisation and real-time simulation of urban systems	Trust, symbolic route transformation and communication history are often external to the computational state

3. Materials and Methods

All figures, equations, tables and methodological materials used in this study were created by the author; no third-party permissions were required.

3.1. Model entities, signals and field state

Let a controversy or communication system be represented by a directed graph

$$G = (V, E), \tag{1}$$

where V contains actors, organisations, institutions or analytically defined groups and E contains directed communication relations. A signal m is a claim, image, statistic, testimony, correction or institutional announcement. A frame is an organisation of meaning within a signal. The Symbolic Mediation Field is the joint state of actor orientations and relational transporters through which signals propagate.

In an urban application, nodes may represent residents, neighbourhood groups, utilities, local authorities, technology firms, media organisations or AI-enabled decision services. An AI system is treated as a source or decision node only when it emits recommendations, rankings, forecasts or explanations that enter a social interaction; the model does not anthropomorphise the software.

Each node i carries an issue-specific complex state

$$\psi_i(t) = r_i(t)e^{i\theta_i(t)}, \qquad r_i(t) \geq 0, \tag{2}$$

where r_i denotes the strength, confidence or salience of the orientation and θ_i its interpretive orientation relative to a local reference. These are latent constructs, not literal wavefunctions.

A directed link from j to i has two components:

$$K_{ij}(t) \geq 0, \qquad U_{ij}(t) = e^{-iA_{ij}(t)} \in U(1). \tag{3}$$

K_{ij} is the exposure or channel strength. U_{ij} is the symbolic transporter, and A_{ij} is the accumulated relational phase associated with source identity, credibility, medium and prior interaction. The field state is

$$\Phi(t) = \{\psi_i(t),\ K_{ij}(t),\ U_{ij}(t) : (i,\ j) \in E\}. \tag{4}$$

Each actor or group also has an issue-specific signed coupling coefficient $q_i \in \mathbb{R}$. Its magnitude represents responsiveness to the field; its sign represents alignment or counter-alignment. Social charge is therefore not affective intensity. Emotion may influence q_i empirically, but the constructs are distinct.

3.2. QSD postulates

The formal postulates and their sociological interpretations are summarised in Table 2.

Table 2. Formal Postulates and Sociological Interpretation of QSD.

Postulate	Formal claim	Sociological implication
P1: Field mediation	Signal effects are functions of Φ(t), not content alone.	Identical messages can generate different responses in different relational states.
P2: Signed local coupling	q_i varies in magnitude and sign.	The same signal may move groups in opposite directions.
P3: Gauge covariance	Arbitrary local phase origins do not alter observables.	Conclusions cannot depend on coding baselines chosen by the analyst.
P4: Dynamic transport	$U_{ij}(t)$ is endogenous.	Institutional events can change later message effects without changing network topology.
P5: Route memory	Transport composes along ordered paths.	Source sequence can matter after message content is held constant.
P6: Loop curvature	Closed routes can accumulate non-zero holonomy.	Contradictory credibility cycles can persist as meso-level structure.
P7: Comparative nesting	Simpler influence models are limiting cases.	Additional QSD structure is retained only when it improves explanation or prediction.

4. Gauge-Covariant Computational Method

4.1. Local re-description and invariance

Let $\phi_i \in \mathbb{R}$ denote an arbitrary local change in the phase origin used to represent node i. QSD requires covariance under

$$\psi_i' = e^{i\phi_i}\psi_i, \qquad U_{ij}' = e^{i\phi_i}U_{ij}e^{-i\phi_j}, \qquad J_i' = e^{i\phi_i}J_i. \tag{5}$$

Here J_i is an external symbolic excitation applied at node i. Writing $\phi_i = q_i\chi_i$ recovers the familiar charge-weighted QED notation, but the empirical coupling coefficient q_i is retained explicitly in the response dynamics below. This separation prevents an arbitrary representation choice from being mistaken for an estimated social susceptibility.

4.2. Gauge-invariant observables

The local magnitude is

$$\rho_i = |\psi_i|^2 = r_i^2. \tag{6}$$

Relational alignment is

$$C_{ij} = Re\left(\psi_i^* U_{ij} \psi_j\right) = r_i r_j \cos\left(\theta_j - \theta_i - A_{ij}\right). \tag{7}$$

For a directed path $\mathcal{P}: s \to i$, define

$$U(\mathcal{P}) = \prod_{(jk)\in\mathcal{P}} U_{jk}, \qquad C_{\mathcal{P}} = Re\left[\psi_i^* U(\mathcal{P}) \psi_s\right]. \tag{8}$$

For a closed cycle $\mathcal{C}$,

$$W(\mathcal{C}) = \prod_{(ij)\in\mathcal{C}} U_{ij} = e^{-i\Omega_{\mathcal{C}}}, \qquad \Omega_{\mathcal{C}} = \sum_{(ij)\in\mathcal{C}} A_{ij} (mod\ 2\pi). \tag{9}$$

$W(C)$ is the Wilson-loop analogue and Ω_C is relational curvature. A non-zero value represents transformation accumulated around a closed communication route that cannot be removed by changing local coding origins.

Figure 1 illustrates gauge-covariant symbolic transport across alternative communication routes and the closed-loop quantity used to diagnose route inconsistency.

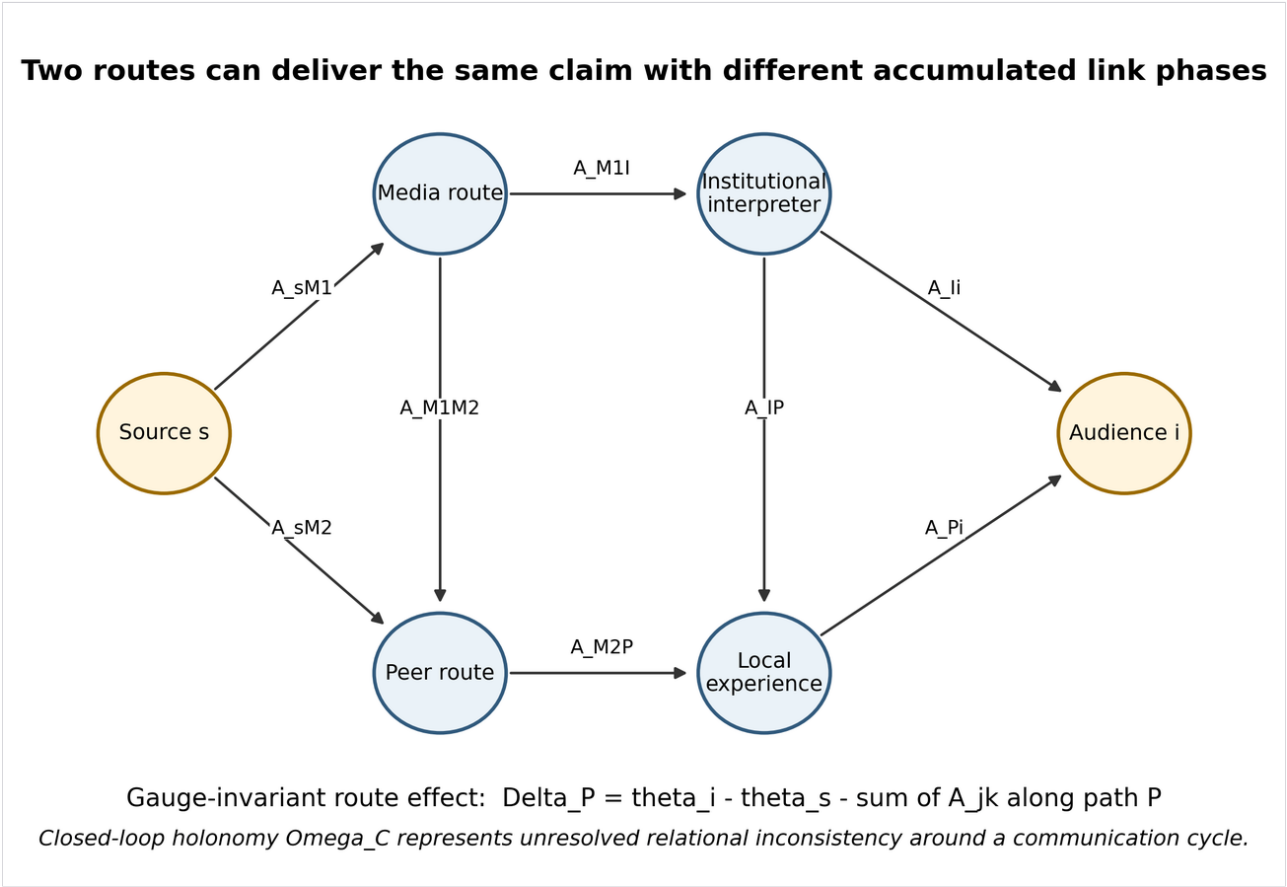

Figure 1. Gauge-Covariant Symbolic Transport Across Alternative Communication Routes. Different routes can accumulate distinct relational phases; closed loops identify unresolved relational inconsistency. Source: Author-created figure.

4.3. Field functional

A minimal gauge-invariant field functional is

$$\mathcal{F}_{QSD} = \sum_{i\in V} V_i\left(|\psi_i|^2\right) + \sum_{(i,j)\in E} K_{ij}|\psi_i - U_{ij}\psi_j|^2 + \kappa\sum_{\mathcal{C}}\left[1 - \cos\left(\Omega_{\mathcal{C}}\right)\right] - \sum_i \left(J_i^*\psi_i + J_i\psi_i^*\right). \tag{10}$$

V_i is a local stability potential, the edge term measures transported mismatch, the loop term penalises curvature and J_i represents external intervention. Equation (10) is a model class rather than a literal universal law. Alternative potentials and update processes can be compared empirically without changing the gauge structure.

4.4. Actor–field coevolution

A dissipative dynamics is

$$\tau_i \frac{d\psi_i}{dt} = -\frac{\partial \mathcal{F}_{QSD}}{\partial \psi_i^*} + q_i J_i(t) + \xi_i(t), \tag{11}$$

$$\tau_{ij} \frac{dA_{ij}}{dt} = -\frac{\partial \mathcal{F}_{QSD}}{\partial A_{ij}} + I_{ij}(t) + \zeta_{ij}(t), \tag{12}$$

where I_{ij} represents institutional or communicative events that directly reconfigure a relation and ξ_i, ζ_{ij} are disturbances. Equation (11) updates orientations; Equation (12) updates the symbolic medium. Their joint use is the formal expression of field memory.

For panel, network or experimental data, a discrete version is

$$\tilde{\psi}_i(t+1) = \alpha_i\psi_i(t) + q_i\sum\nolimits_j \beta_{ij}K_{ij}(t)U_{ij}(t)\psi_j(t) + q_iJ_i(t), \tag{13}$$

$$\psi_i(t+1) = \frac{\tilde{\psi}_i(t+1)}{\max\left(1, \left|\tilde{\psi}_i(t+1)\right|\right)}, \tag{14}$$

and

$$A_{ij}(t+1) = \left(1 - \lambda_{ij}\right)A_{ij}(t) + \lambda_{ij}\hat{A}_{ij}(t) + \eta_{ij}I_{ij}(t). \tag{15}$$

The parameter λ_{ij} controls relational memory. Small values imply persistent link states; large values imply rapid updating. The covariant update makes three quantities empirically separable: exposure magnitude K_{ij}, symbolic transformation U_{ij} and signed response q_i.

4.5. AI implementation workflow

A practical implementation begins by constructing a directed source–audience graph from communication logs, institutional records, surveys or platform data. Actor states ψ_i are estimated from issue-specific belief, trust, risk or acceptance indicators; exposure weights w_{ij} are estimated from observed communication frequency or attention; signed couplings q_i are estimated from heterogeneous treatment response or longitudinal sensitivity; and link phases A_{ij} are estimated from source-conditioned deviations between message content and observed response.

The computational sequence is: (1) ingest time-stamped signals and events; (2) update actor, exposure and link states; (3) calculate gauge-invariant edge alignment, path transport and loop holonomy; (4) supply these quantities to an urban digital twin, agent-based simulator, machine-learning model or decision dashboard; (5) generate counterfactual communication routes; and (6) compare the full method with flat-link, homogeneous-coupling and fixed-field ablations.

For a sparse graph G = (V, E), a single actor–link update is O(|E|). Holonomy can be evaluated on a chosen fundamental cycle basis, with cost proportional to the summed lengths of those cycles. The method is therefore computationally tractable for large but sparse urban communication networks, subject to the cost of estimating latent states from data.

All figures and tables in this article were created by the author. No third-party images, proprietary software, questionnaires or restricted tools were used, and no permissions are required for the proposed method.

5. Results

5.1. Theorem 1: gauge invariance

Theorem 1. Under the local transformation in Equation (5), the quantities ρ_i, C_{ij}, $C_{\mathcal{P}}$, $W(\mathcal{C})$ and $\mathcal{F}_{QSD}$ are invariant.

Proof sketch. The node magnitude satisfies $\left|e^{i\phi_i}\psi_i\right|^2 = |\psi_i|^2$. In the edge observable, the phase factors cancel:

$$\left(\psi_i'\right)^* U_{ij}'\psi_j' = \psi_i^* e^{-i\phi_i}\left(e^{i\phi_i}U_{ij}e^{-i\phi_j}\right)e^{i\phi_j}\psi_j = \psi_i^* U_{ij}\psi_j. \tag{16}$$

The cancellation telescopes along a path, leaving only endpoint phases, which then cancel against the transformed endpoint states. On a closed cycle every endpoint is also an intermediate point, so all local factors cancel and $W(\mathcal{C})$ is unchanged. Each term in Equation (10) is therefore invariant. A complete derivation is provided in Supplementary file 1, Section S1.

The theorem gives the social meaning of gauge invariance: substantive conclusions depend on relations among states and links, not on arbitrary local labels chosen by communities or analysts.

5.2. Theorem 2: flatness and path independence

Theorem 2. Let G be connected. Symbolic transport is path independent between every ordered pair of nodes if and only if

$$W(\mathcal{C}) = 1 \quad \text{for every closed cycle } \mathcal{C}. \tag{17}$$

Equivalently, a flat field admits node phases γ_i such that

$$U_{ij} = e^{i\gamma_i}e^{-i\gamma_j} \qquad \text{for all } (i,\ j) \in E. \tag{18}$$

Proof sketch. If path transport is independent of route, traversing any path and returning along another must yield the identity, so every cycle holonomy is one. Conversely, if every cycle holonomy is one, choose a root node and define $e^{-i\gamma_i}$ as transport from the root to i. Zero cycle holonomy guarantees that this definition is independent of the selected root-to-node path, yielding Equation (18). Supplementary file 1, Section S1, provides the graph-theoretic construction.

Corollary 1. Non-zero holonomy is an irreducible marker of route dependence. It cannot be eliminated by relabelling local interpretive baselines.

This result is central to QSD's novelty. A standard influence matrix may assign different weights to different routes, but QSD identifies whether those differences are removable representation choices or a structural inconsistency preserved by every admissible re-description.

5.3. Proposition 1: the classical influence limit

Proposition 1. Suppose: (i) $U_{ij} = 1$ for all edges; (ii) $\psi_i = x_i \in \mathbb{R}$; (iii) $q_i \geq 0$; (iv) link states are fixed; (v) the normalisation in Equation (14) is inactive; and (vi) coefficients are row-normalised. Then Equation (13) reduces to a weighted linear influence model. With a fixed predisposition term it yields the Friedkin–Johnsen form.

In vector notation,

$$\mathbf{x}(t+1) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{x}(0) + \mathbf{j}(t). \tag{19}$$

Thus QSD does not discard the classical influence tradition. It contains it as the flat, real-valued, static-link limit. The empirical burden is correspondingly clear: dynamic transport, signed coupling or loop curvature should be retained only when they explain patterns that Equation (19) cannot.

5.4. Proposition 2: ordered exposure from field updating

Let $\mathcal{T}_m$ denote the joint update of actor and link states produced by message m. In general,

$$\mathcal{T}_{m_2} \circ \mathcal{T}_{m_1}(\Phi) \neq \mathcal{T}_{m_1} \circ \mathcal{T}_{m_2}(\Phi) \tag{20}$$

whenever at least one message changes U_{ij}, K_{ij} or the non-linear normalisation regime. The order effect is therefore not asserted as mysterious indeterminacy; it follows from coevolution. A claim followed by a correction is processed through a field already changed by the claim, whereas the reverse sequence is processed through a different field.

5.5. Testable technical predictions

The model yields five primary predictions.

1. **Signed response:** identical exposure can move groups in opposite directions when $q_i q_j < 0$.
2. **Route dependence:** paths with the same endpoints can produce different alignments when the enclosed holonomy is non-zero.
3. **Field memory:** an institutional event changes later responses to unchanged messages through persistent $A_{ij}(t)$.
4. **Curvature instability:** networks with larger aggregate loop curvature should display lower cross-route credibility consistency and slower convergence.
5. **Classical collapse:** when estimated link phases are flat and static, QSD should collapse empirically to a conventional influence model without predictive loss.

6. Computational Testing

6.1. Test design

The computational tests evaluate internal entailment rather than population-level predictive accuracy. They ask whether the proposed update rules are sufficient to produce the distinctive patterns claimed by the method and whether those patterns disappear under structural ablation.

The deterministic simulations test whether the stated mechanisms generate the patterns claimed by the theory. They are demonstrations of mathematical entailment, not estimates of population effects. Three audience groups receive a contested claim $C = e^{i\pi}$ and a correction $K = 1$. The update uses Equations (13)–(15) with $\alpha = 0.55$, $\beta = 0.75$ and link adaptation $\eta = 0.90$.

The initial conditions for the deterministic order-and-coupling test are reported in Table 3.

Table 3. Initial Conditions for the Order-and-Coupling Simulation.

Group	Initial state $\psi_i(0)$	Initial link phase $A_i(0)$	Coupling q_i
Receptive	$0.45e^{i0.10}$	0.15	1.00
Ambivalent	$0.25e^{i0.80}$	0.70	0.45
Oppositional	$0.50e^{i2.60}$	1.10	−0.80

6.2. Coupling and order

To evaluate order effects without relying on gauge-dependent components of the latent state ψ_i, the simulation was reanalysed using the normalised transported alignment $R_i = \mathrm{Re}(\psi_i^* U_i K\, K)/(|\psi_i||K|)$, with the correction proposition $K = 1$ held as the same substantive reference in both sequence conditions. This fixed reference is necessary because claim → correction and correction → claim end with different messages; evaluating each condition against its last message would compare different outcome objects rather than the effect of ordering on a common proposition. R_i lies in [−1, 1], with positive values denoting transported alignment with the correction and negative values denoting counter-alignment. An order reversal is present when $R_i(C{\to}K) \times R_i(K{\to}C) < 0$.

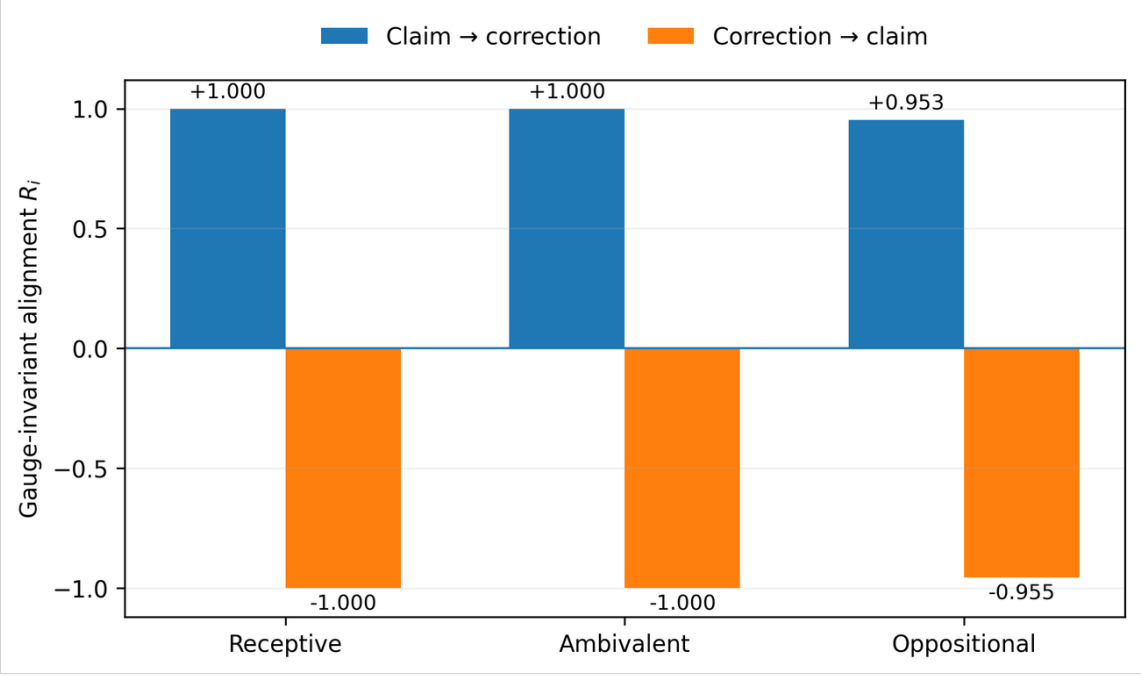

Figure 2. Gauge-Invariant Order Effects in the QSD Simulation. Normalised transported alignment R_i is evaluated against the same correction reference (K = 1) after claim → correction and correction → claim. Positive values indicate alignment and negative values counter-alignment. Values are model-generated rather than empirical estimates. Source: Author-created figure.

Figure 2 shows a large sequence-dependent reversal relative to the fixed correction reference. Claim → correction produces R_i = 1.000, 1.000 and 0.953 for the receptive, ambivalent and oppositional groups, respectively; correction → claim produces −1.000, −1.000 and −0.955. The corresponding order contrasts, $\Delta_i(\mathrm{order}) = R_i(C{\to}K) - R_i(K{\to}C)$, are 2.000, 2.000 and 1.908, and the reversal products are −1.000, −1.000 and −0.910. The oppositional-group reversal therefore survives the gauge-invariant reanalysis: its initial alignment with the correction is R_i = −0.848, but the final value is +0.953 after claim → correction and −0.955 when the order is reversed.

The result is, however, narrower than the interpretation based on the real component of an individual complex state. Reversal is not unique to the negatively coupled group: under the strong link-adaptation setting used in this demonstration ($\lambda = 0.90$), all three groups cross zero when sequence is reversed. The simulation therefore demonstrates gauge-invariant order dependence and field-mediated history effects, not that negative coupling by itself is sufficient to produce directional reversal. Signed coupling remains a source of heterogeneous response, but its independent contribution must be established by ablation or parameter variation rather than inferred from the sign of an individual state component.

As a numerical invariance check, arbitrary local phase re-descriptions were applied to the final states and links. R_i remained unchanged to floating-point precision (maximum absolute deviation 5.55×10^{-16}), whereas $\mathrm{Re}(\psi_i)$ is gauge dependent. The conclusion in Figure 2 therefore does not depend on the local phase convention used to represent actor states. If alignment is instead defined relative to whichever message arrives last, both sequences end positively aligned with their respective final message; the reversal reported here is specifically a route-history effect measured against the same fixed substantive reference.

6.3. Institutional shock and field memory

A second simulation repeats the same positive message while introducing a shock to the source–audience link at step 6. The link phase changes from 0.20 to 1.60 radians and relaxes toward baseline with $\lambda = 0.12$.

Figure 3 shows the simulated persistence and gradual recovery of the relational field after an institutional shock.

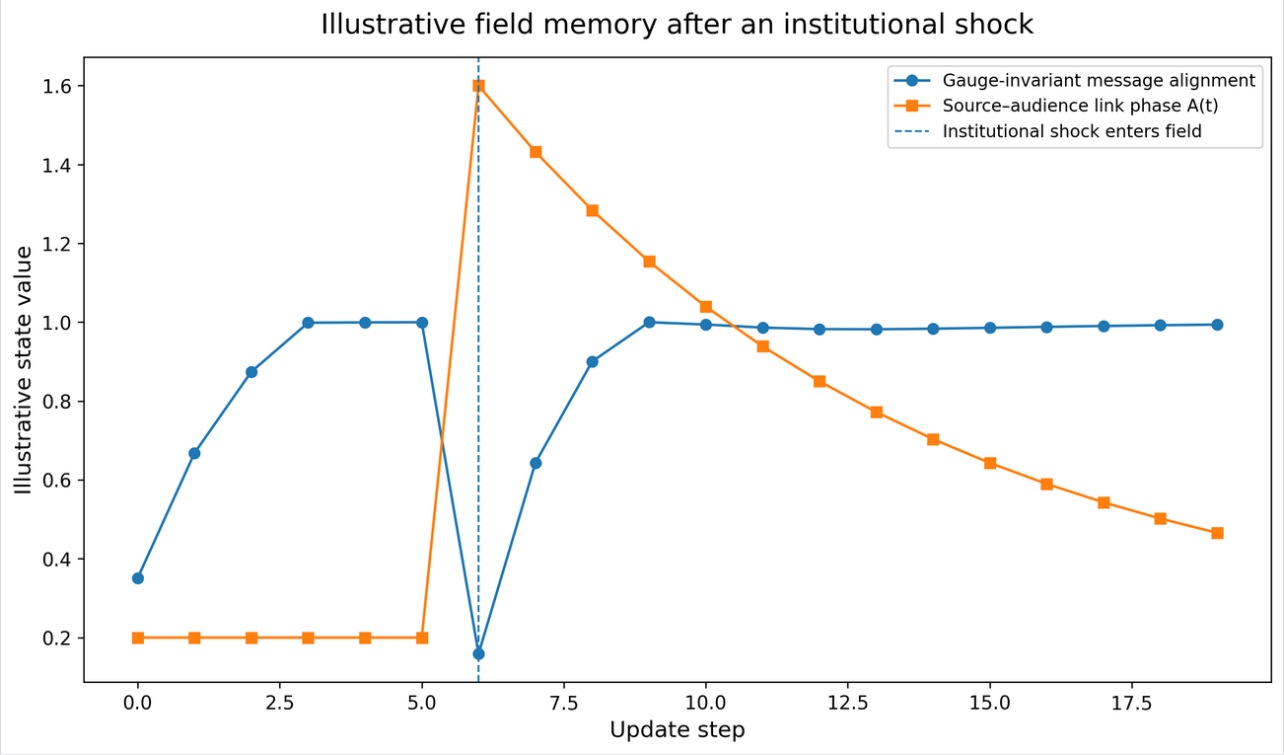

Figure 3. Field Memory Following an Institutional Shock. The relational phase changes at step 6 and relaxes gradually while message alignment recovers. Source: Author-created figure.

Gauge-invariant alignment is 0.9998 immediately before the shock and 0.1591 immediately after it. It recovers gradually to 0.9825 by step 12. Content is held constant; the temporary response change is attributable to the altered link field.

6.4. Structural ablations

Table 4 summarises the structural ablations and the consequences predicted when each QSD component is removed.

Table 4. Structural Ablations and Their Predicted Consequences.

Ablation	Removed structure	Predicted consequence
$q_i = q > 0$ for all i	Signed heterogeneous coupling	Directional heterogeneity attributable specifically to signed coupling is removed; order effects may persist through field updating
$U_{ij} = 1$	Symbolic transport phase	Route transformation and holonomy disappear
$A_{ij}(t + 1) = A_{ij}(t)$	Link adaptation	Institutional shocks cannot create persistent field memory
$W(C) = 1$ for all cycles	Curvature	Transport becomes path independent by Theorem 2
Real, static, row-normalised system	Full gauge structure	Model reduces to Equation (19)

These ablations define the comparative standard. If data show no loss when the added structures are removed, QSD is unnecessary for that application.

7. Sustainable-City Application: Urban Heat Decarbonisation

7.1. Application design

A purposive corpus of 27 claim-records from 19 UK-relevant public documents was used as a sustainable-city application. Domestic heat decarbonisation is an urban infrastructure transition involving buildings, electricity networks, household affordability, installers, local experience and public policy. The corpus is not a prevalence sample, records are not independent, coding was single-researcher and no audience outcomes were measured. It therefore illustrates construct implementation rather than validating the gauge model.

The source documents included fact-checks, news reports, opinion pieces, advocacy material, advertising controversies and consumer guidance. Records were coded for claim status, symbolic carrier, fear or grievance, institutional target, anticipated future and inferred trust implication. These fields provide candidate inputs for an AI-assisted communication-risk monitor or urban digital-twin module, but they require independent coding and audience validation before operational deployment.

7.2. Recurrent symbolic configurations

Figure 4 summarises the three recurrent symbolic configurations identified in the bounded heat-pump application.

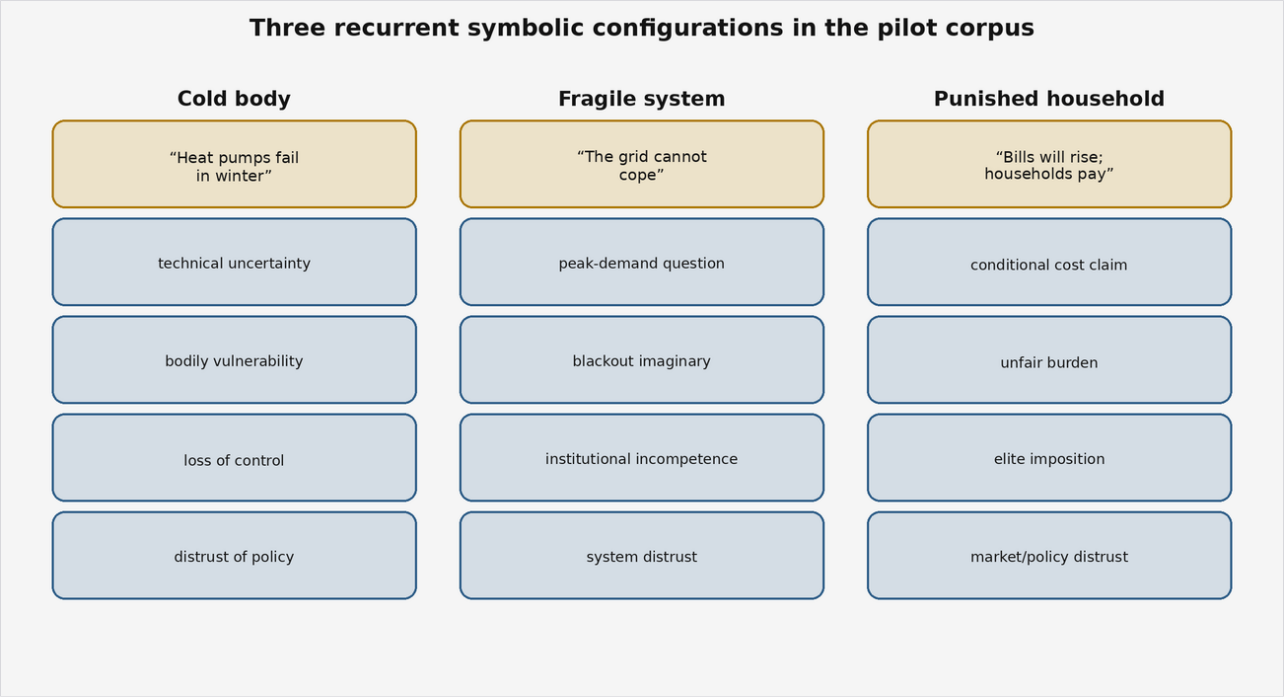

Figure 4. Recurrent Symbolic Configurations in the Heat-Pump Application. The corpus yields cold-body, fragile-system and punished-household pathways. Source: Author-created figure.

The cold body links a performance claim to bodily vulnerability, loss of control and policy distrust. The fragile system links a peak-demand question to blackout imagery, institutional incompetence and system distrust. The punished household links a conditional cost claim to unfair burden, elite imposition and market or policy distrust. In an AI-supported urban decision system, these configurations could be treated as structured relational features rather than as sentiment labels alone.

These are not merely negative frames. In QSD terms they are candidate local field configurations: recurrent conjunctions of signal content, vulnerability, source attribution and anticipated future that may support different coupling patterns across groups. They can be converted into experimental treatments and linked to measured source relations and exposure routes.

7.3. Application results and boundaries

The application supports three bounded conclusions. First, QSD variables can organise real urban-transition communication without reducing every practical concern to misinformation. Second, technically similar claims can occupy different source, vulnerability and future-oriented structures. Third, the coding produces testable inputs for route randomisation, source-specific prediction and longitudinal field updating. The application does not establish population prevalence, persuasion, trust change, field curvature or superiority over classical or machine-learning models.

8. AI Integration and Validation

QSD is intended to complement, not replace, existing artificial-intelligence methods. Its role is to create interpretable relational features and counterfactual scenarios for systems that forecast adoption, optimise communication, evaluate policy risk or simulate urban transition pathways. Operational use requires transparent estimation, uncertainty reporting, data-governance controls

and human oversight. This requirement is consistent with recent human-centric AI and smart-building research, which emphasises explainability, integration of subjective and objective data, data governance and occupant or stakeholder engagement as conditions for responsible deployment (Aziz *et al.* 2026; Onweh *et al.* 2025; Razak *et al.* 2025).

8.1. Estimation and feature engineering

Empirical estimation requires repeated measures of actor state, source relation and exposure history. A possible observation model is

$$\mathbf{y}_i(t) = \mathbf{H}\mathbf{s}_i(t) + \epsilon_i(t), \qquad \mathbf{s}_i(t) = \begin{bmatrix} Re\psi_i(t) \\ Im\psi_i(t) \\ A_{i1}(t) \\ \vdots \end{bmatrix}, \tag{21}$$

where $\mathbf{y}_i(t)$ contains observed belief, trust, perceived risk, fairness, intention or behaviour. Link phases can be estimated from source-specific treatment responses, longitudinal credibility ratings and route-randomised exposure.

8.2. Validation design

A decisive test should randomise: (i) message content; (ii) source identity; (iii) order; and (iv) prior field history. Primary outcomes should be measured before exposure, immediately afterwards and at delayed follow-up. Route dependence can be tested by delivering identical final information through different source sequences. Field memory can be tested by exposing groups to different institutional histories before giving them an identical final signal.

8.3. Model comparison

QSD should be compared with:

- DeGroot and Friedkin–Johnsen influence models;
- network-autocorrelation and state-space models;
- dynamic-network diffusion models;
- conventional moderation and mediation models;
- signed-network or structural-balance models;
- Bayesian trust-learning models; and
- flexible machine-learning benchmarks.

The comparison should use held-out prediction, calibration, information criteria and ablation. In-sample fit is insufficient because QSD contains more structure than a scalar influence model.

8.4. Rejection criteria

QSD should be reduced or rejected for a given domain if:

1. q_i cannot be distinguished from ordinary prior attitude;
2. estimated U_{ij} adds no predictive information beyond scalar weights;
3. route effects disappear after source trust and exposure are controlled;
4. link history adds no delayed predictive value;
5. holonomy cannot be estimated reliably or has no relation to cross-route inconsistency;
6. the flat classical limit predicts held-out data equally well; or
7. independent replications fail.

These criteria make the theory vulnerable to evidence while preserving the formal priority claim. Novelty and empirical adequacy are separate questions.

9. Discussion

9.1. Contribution to AI and sustainable cities

QSD contributes to artificial intelligence for sustainable cities by formalising a variable that is normally left outside the urban model: the transformation of meaning and trust along communication routes. This distinction separates channel strength from symbolic transport and allows a city system to be topologically stable yet relationally reconfigured after policy reversals, service failures, misleading promotion or trusted local demonstrations.

Theorem 2 provides an interpretable diagnostic. In a flat field, link phases can be removed by local re-description and the additional QSD structure is unnecessary. In a curved field, route dependence is irreducible. Holonomy therefore identifies when apparently inconsistent community responses cannot be explained solely by different coding baselines or exposure weights.

For sustainable-city applications, the principal outputs are credibility communities, fragmented trust systems, source-sensitive adoption patterns, message-routing risks and persistent institutional effects. These outputs can augment predictive models and digital twins by indicating why an intervention may fail socially even when it is technically optimal.

9.2. Why the QED derivation is substantive

The QED derivation contributes more than terminology. It imposes a discipline of local covariance, requires a distinction between states and mediators, identifies invariant observables, and yields loop quantities with a defined mathematical role. Without those constraints, “field”, “charge” and “mediation” could remain loose metaphors. With them, the theory specifies which quantities can change under recoding and which cannot.

The theory remains an effective social model rather than a physical ontology. QED motivates the architecture; social meaning and empirical estimation determine whether the architecture is useful. This position avoids both reductionism and decorative physics.

9.3. Relation to existing computational models

QSD is complementary to existing computational approaches. When phases are absent, links are static and states are real, it reduces to weighted influence. When only link signs matter, it approaches structural-balance models. When link history is central, it connects with coevolving-network methods. It can also be used upstream of machine learning as a gauge-invariant feature layer or downstream as an interpretability and counterfactual-analysis module.

This nesting matters methodologically. QSD does not claim that every smart-city problem requires complex phases or holonomy. The additional structure is justified only when identical content has source- or route-dependent effects, when institutional events alter later response, or when closed communication cycles contain persistent inconsistency that scalar features cannot remove.

9.4. Scope and limitations

The article establishes formal novelty, invariance properties, limiting cases and computational entailment. It does not claim population-level validation. The heat-pump corpus is small and illustrative; the simulations are deterministic; the latent-state interpretation requires measurement development; and heterogeneous coupling may be difficult to identify without randomised source and route variation.

Further mathematical work should study existence, stability and convergence under alternative potentials; identifiability of q_i and A_{ij}; estimation on sparse and multiplex networks; non-Abelian extensions for multidimensional meaning; and statistical tests for aggregate curvature. Those

extensions are consequences of an already specified architecture, not substitutes for the novelty claimed here.

The current tests establish internal consistency, limiting behaviour and computational proof of concept. They do not establish external validity, operational safety or fairness in live AI systems. Before deployment, QSD requires preregistered experiments, independent coding, parameter-recovery analysis, held-out comparison with simpler models, bias assessment and governance procedures that prevent the optimisation of manipulative communication.

10. Conclusion

Quantum Sociodynamics introduces a gauge-covariant computational method for representing symbolic influence and public trust in AI-supported sustainable-city transitions. The Symbolic Mediation Field separates actor orientation, source-conditioned transport, exposure strength, signed coupling and institutional history. Gauge invariance protects substantive outputs from arbitrary local coding origins; the flatness result distinguishes removable representation from irreducible route inconsistency; and the classical limit shows when standard influence models are sufficient.

The novelty is specific and technically testable. QSD is not the first use of fields, complex numbers, quantum ideas, trust models or artificial intelligence in social analysis. It is, to our knowledge, the first method to integrate a state-bearing symbolic mediator field, signed coupling, local gauge covariance, dynamic source–audience transport, route and loop observables, field feedback and a classical influence limit for AI-assisted urban-transition analysis. The simulations and heat-pump application demonstrate implementation; subsequent studies must determine whether the added structure improves prediction, interpretation and responsible decision support.

Declarations

Author contributions. The sole author conceived the theory, developed the mathematical formulation, designed and interpreted the computational demonstrations, assembled and coded the pilot corpus, prepared the figures and wrote the manuscript. Permissions All figures, tables, equations and methodological materials were created by the author. No third-party images or proprietary research instruments were reproduced, and no external permissions were required.

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Conflicts of interest. The author has no competing interests to declare.

Data availability. Supplementary file 1 contains the formal derivations, computational specification, complete pilot-corpus inventory, claim-record map and QSD construct codebook. Supplementary file 2 contains the structured pilot claim-record data. Supplementary file 3 contains the simulation parameters. Supplementary file 4 contains the ablation definitions. Figures 1–4 are supplied separately as 300 dpi PNG files. No private human-participant data were collected. The underlying public documents remain available from their original publishers.

Ethics. The study involved no human participants, human tissue, animals or private personal data. The bounded application used publicly accessible organisational and media documents and did not identify private individuals. Formal ethics approval and informed consent were therefore not applicable.

Generative AI. AI tools were used for language editing, document formatting and organisational assistance. It was not used to fabricate data, generate unverifiable references or determine the scholarly conclusions. The author verified the sources, equations, calculations, interpretations and final text and accepts full responsibility for the manuscript.

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