

Analytical Solution of Seepage Flow in a Confined Aquifer with a Standing Column Well

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Abstract: The standing column well for ground source heat pump systems is a promising technology with high efficiency and environmental benefit, where groundwater is drawn from the bottom of a well and then re-injected to its top after transferring heat with heat pumps. Heat transfer analysis of the ground and aquifer involves complex problems in such a case. Determining the groundwater seepage flow is a precondition to solve the energy equation describing the heat transfer of the system. Only when piezometric head is obtained, the seepage velocity can be determined according to Darcy's law. This paper studies the groundwater seepage flow in an axial symmetrical geometry with the assumption that gross groundwater flow is neglected. An analytical solution of the groundwater seepage flow for a confined aquifer has been acquired by the integral transform method, which may provide a foundation for heat transfer analysis of the standing column well system.

Keywords: Ground source heat pump, Standing column well, Seepage flow, Heat transfer

1. INTRODUCTION

A number of authors have reported on the geothermal heat pump system with a standing column well in the last decade [1-3]. According to the ASHRAE Handbook Standing Column Well (SCW) system is looked upon as a main kind of Ground Source Heat Pump (GSHP) systems. The standing column wells are feasible in areas with near surface bedrock. In SCW systems, the ground heat exchanger consists of one or more wells, typically about 400 m to 500 m deep and about 15 centimeters in their diameter. Groundwater is pumped from the bottom, runs through heat pumps and discharged at the top of the well in this system. Figure 1 illustrates the groundwater flow in and around a SCW. The system is of traits of both groundwater heat pumps (GWHPs) and ground-coupled heat pumps (GCHPs). Its heat transfer efficiency is superior to the GCHP system, and there isn't groundwater waste and trouble of groundwater re-injection such as in GWHP systems with multi-wells. So SCWs can be used as ground heat exchangers with high efficiency and environmental benefit in GSHP systems, where hydrological and geological conditions are suitable. And it can also constitute a multiform and integrated system with other kinds of ground source heat pump technologies to meet the heating and cooling requirements in different applications.

When a SCW serves as the ground heat exchanger in GSHP systems, groundwater in the well acts just as the heat transfer medium. The quantity of heat that the heat pumps acquire comes from the aquifer, and the function of the well in the heat transfer process is similar to that of boreholes in the vertical ground-loops. The main difference between the two systems is that the groundwater, i.e. the heat transfer medium, touches directly the rock and soil in the SCW

systems. Both the radial and longitudinal flow of groundwater must be thought about in such a case so that it is a complex multi-dimensional seepage flow with all complexities in geological and hydraulic conditions. The heat transfer analysis with the SCW is even harder than that with GCHPs, and it involves seepage flow and heat transfer problems in porous media. Some researchers have developed a numerical model of the groundwater flow and heat transfer both

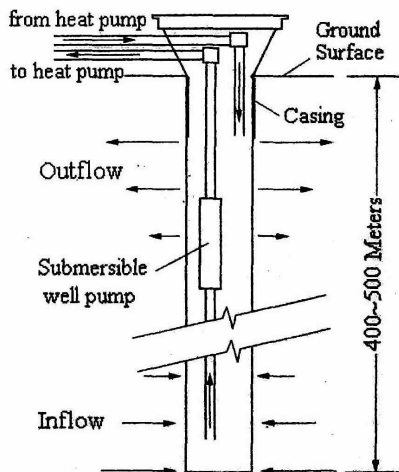


Fig. 1: A diagram showing SCW geothermal heat exchanger

within the well and in the surrounding aquifer.^[4]

The seepage flow problem in the porous aquifer must be solved firstly to analyze heat transfer because the energy equation in such a system includes convection terms. The convective heat transfer by groundwater flow usually is the most significant heat transfer mechanism in such cases. The seepage velocity can easily be obtained according to the Darcy's law as long as the piezometric head is determined.

Aquifers are described as being either confined or phreatic/unconfined^[5-7]. Confined aquifers are bounded between two or more layers of rock (or clay soils) of low permeability. Phreatic/unconfined aquifers are bounded at their upper surface by the water table. In this study, we have developed a mathematical model of piezometric head of the groundwater in the confined aquifer and derived an analytical solution for the piezometric head by integral transformation method, which provides a foundation for heat transfer analysis of the SCW systems.

2. MATHEMATICAL MODEL

Figure 2 shows the seepage flow around a standing column well on the condition that the aquifer is confined at both its top and bottom. Water is recirculated from one end of the column to the other. In order to intensify heat transfer between the circulating water and aquifer, in general, a barrier may be set in the well so that the water is forced to flow into the surrounding aquifer and then drawn into the well from its lower section. Gross groundwater flow in the lateral direction arising from head gradients induced by adjacent rivers and so on has not been considered in this study. It can be seen from Figure 2 that this is a radial-axial geometry problem. Assumptions about the seepage flow in the confined aquifer have been made to simplify the theoretical analysis as usually accepted in literatures^[5-7].

1. The aquifer is homogeneous but anisotropic, spreads to infinity in lateral direction with uniform thickness. The coordinate axes accord with the principal directions of its hydraulic conductivity.
2. The pumping flow rate is constant and equal to the re-injection rate. And a blockage is set in the midline of the well and serves as the interface between pumping part and re-injection part of the well.
3. Seepage fluxes are distributed uniformly along the z-direction on the well wall of both pumping and re-injection sections.
4. The well is fully penetrating, and its radius is assumed to be infinitely small.
5. Saturated flow is assumed in the aquifer, and so Darcy's law is obeyed.
6. The hydraulic gradient remains zero throughout the domain concerned before the SCW system operating.

On basis of above assumptions the equation of seepage flow in terms of the drawdown is given by

$$K_r \left(\frac{\partial^2 S}{\partial r^2} + \frac{1}{r} \frac{\partial S}{\partial r} \right) + K_z \frac{\partial^2 S}{\partial z^2} = \mu_s \frac{\partial S}{\partial \tau} \quad (1)$$

where K_r and K_z are the principal hydraulic conductivities in the r-direction and z-direction respectively, S is the drawdown, μ_s is the specific storativity, and τ is the time.

According to the knowledge of dynamics of groundwater the r-direction boundary conditions can

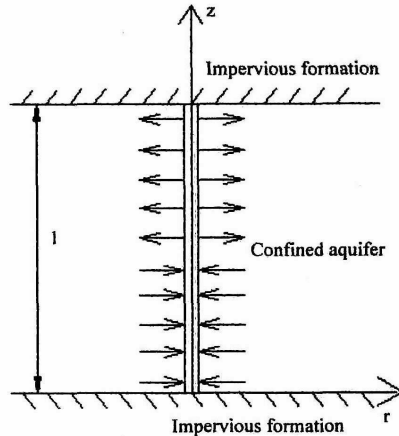


Fig. 2: A diagram showing a fully penetrating well in confined aquifer with a SCW

be given as follow^[5-7].

$$\lim_{r \rightarrow 0} r \frac{\partial S}{\partial r} = -\frac{Q}{\pi K_r}, \quad 0 \leq z \leq \frac{l}{2} \quad (2)$$

$$\lim_{r \rightarrow 0} r \frac{\partial S}{\partial r} = \frac{Q}{\pi K_r}, \quad \frac{l}{2} < z \leq l$$

$$r \rightarrow \infty, S = 0 \quad (3)$$

where Q is the total discharge flow rate, m^3/s , and l is the thickness of the confined aquifer, m.

Because there are horizontal impervious formations on the top and bottom of the aquifer, the z-direction boundary conditions can be given by

$$z = 0, \quad \frac{\partial S}{\partial z} = 0 \quad (4)$$

$$z = l, \quad \frac{\partial S}{\partial z} = 0 \quad (5)$$

And according to the 6th assumption above the initial condition can be written as

$$\tau = 0, S = 0 \quad (6)$$

So equations (1) - (6) consist of the whole mathematical formulation about seepage flow problem in confined aquifer of SCW systems.

3. ANALYTICAL SOLUTION

We have derived the analytical solution of this problem by integral transformation method. Some details are described below.

The first step is to apply Laplace transformation to partial differential equation (1) and related boundary conditions (2) ~ (5), and then we obtain

$$K_r \left(\frac{\partial^2 \bar{S}_L}{\partial r^2} + \frac{1}{r} \frac{\partial \bar{S}_L}{\partial r} \right) + K_z \frac{\partial^2 \bar{S}_L}{\partial z^2} = \mu_s p \bar{S}_L \quad (7)$$

$$r \rightarrow 0, \lim_{r \rightarrow 0} r \frac{\partial \bar{S}_L}{\partial r} = \begin{cases} -\frac{Q}{\pi K_r p} & 0 \leq z \leq \frac{l}{2} \\ \frac{Q}{\pi K_r p} & \frac{l}{2} < z \leq l \end{cases} \quad (8)$$

$$r \rightarrow \infty, \bar{S}_L = 0 \quad (9)$$

$$z = 0, \frac{\partial \bar{S}_L}{\partial z} = 0 \quad (10)$$

$$z = l, \frac{\partial \bar{S}_L}{\partial z} = 0 \quad (11)$$

where $\bar{S}_L(r, z, p) = \int_0^\infty S \cdot \exp(-p\tau) d\tau$ is the Laplace transform of $S(r, z, \tau)$. This results in the time-independent partial differential equation (7) and related boundary conditions (8) ~ (11), which incorporate the initial condition and depend on the Laplace parameter p . As the second step, we apply the finite Fourier transformation to Equations (7) ~ (11) and then obtain

$$\frac{d^2 \bar{S}_{Ln}}{dr^2} + \frac{1}{r} \frac{d \bar{S}_{Ln}}{dr} - N^2 \bar{S}_{Ln} = 0 \quad (12)$$

$$r \rightarrow 0, \begin{cases} \lim_{r \rightarrow 0} r \frac{d \bar{S}_{Ln}}{dr} = -\frac{2Q \sin(n\pi/2)}{n\pi^2 K_r p} & n \neq 0 \\ \lim_{r \rightarrow 0} r \frac{d \bar{S}_{L0}}{dr} = 0 & n = 0 \end{cases} \quad (13)$$

$$r \rightarrow \infty, \bar{S}_{Ln} = 0 \quad (14)$$

$$\text{where } \bar{S}_{Ln}(r, n, p) = \int_0^l \bar{S}_L \cos\left(\frac{n\pi}{l} z\right) dz, n = 0, 1, 2 \dots$$

is the Fourier transform of $\bar{S}_L(r, z, p)$ and

$$N = \sqrt{\frac{K_z}{K_r} \left(\frac{n\pi}{l} \right)^2 + \frac{\mu_s p}{K_r}}. \text{ The result of the transformation, i.e. Equation (12) is a modified Bessel$$

equation of zero order, and Equations (13) and (14) are related boundary conditions.

Obviously, when $n = 0$, the above problem has a meaningless zero solution only, namely $\bar{S}_{L0} = 0$. This is an incidental outcome as we have assumed the interface between pumping and re-injection sections lies in the middle point of the well. When $n \neq 0$, the solution of Equations (12) ~ (14) can be obtained as follows

$$\bar{S}_{Ln}(r, n, p) = \frac{2Q}{n\pi^2 K_r p} \sin\left(\frac{n\pi}{2}\right) K_0(Nr) \quad (15)$$

In turn, conducting the inversion of Fourier transforms, we obtain

$$\bar{S}_L(r, z, p) = \frac{4Q}{l\pi^2 K_r} \sum_{n=1}^{\infty} \left[\frac{\sin(n\pi/2)}{np} K_0(Nr) \cos\left(\frac{n\pi}{l} z\right) \right] \quad (16)$$

And inversion of the Laplace transforms results in

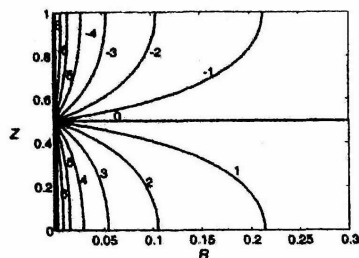
$$S(r, z, \tau) = \frac{2Q}{l\pi^2 K_r} \sum_{n=1}^{\infty} \left[\frac{\sin(n\pi/2)}{n} \times \cos\left(\frac{n\pi}{l} z\right) W\left(\frac{\mu_s r^2}{4K_r \tau}, \sqrt{\frac{K_z}{K_r} \frac{n\pi}{l}}\right) \right] \quad (17)$$

where

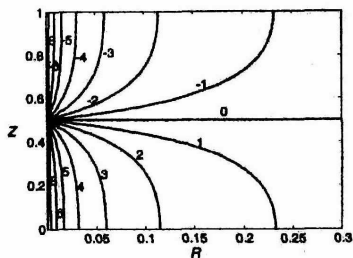
$$W\left(\frac{\mu_s r^2}{4K_r \tau}, \sqrt{\frac{K_z}{K_r} \frac{n\pi}{l}}\right) = \int_{\frac{\mu_s r^2}{4K_r \tau}}^{\infty} \frac{1}{\alpha} \exp\left[-\alpha - \left(\frac{K_z n^2 \pi^2 r^2}{4\alpha K_r l^2}\right)\right] d\alpha \quad (18)$$

is one of well functions in dynamics of groundwater. Equation (17) gives the analytical solution of the original problem described in Equations (1) ~ (6).

Some dimensionless variables are defined to analyze the problem conveniently. $R = r/l$ and $Z = z/l$ are defined as the dimensionless radius and longitudinal coordinate respectively, and $F = 4K_r \tau / (\mu_s l^2)$ is defined as dimensionless time, $D = Sl\pi^2 K_r / (2Q)$ denotes the dimensionless drawdown and $u = \mu_s r^2 / (4K_r \tau) = R^2 / F$. The ratio of hydraulic conductivities, i.e., $K_d = K_z / K_r$, denotes the relative ratio of the two principal hydraulic conductivities.



(a) $K_d=1, F=0.3$



(b) $K_d=0.5, F=0.3$

Fig. 3: The contours of the dimensionless drawdown

The contours of dimensionless drawdown D are shown in Figure 3 when dimensionless time $F = 0.3$, and K_d is fixed as 1 and 0.5 respectively. It's easy to find that the scope of influence of the dimensionless drawdown in Figure3-b is bigger than in Figure3-a.

Moreover, it can be seen from the figure that the hydraulic head is higher near the well than that apart from the well at re-injection section. The situation is just the opposite at the pumping section. Hydraulic head is lowest or highest on the well wall. In addition it can be seen that the impervious boundary conditions are satisfied well at $Z = 0$ and $Z = 1$.

Profiles of the dimensionless drawdown along the r -direction at certain depths ($Z = 0, Z = 1$) are shown in Figure 4. This diagram shows that the drawdown curve looks like an inverse cone at the re-injection part ($Z = 1$), and the drawdown curve looks like a cone of depression at the pumping part of the well ($Z = 0$). In the confined aquifer profiles of drawdown are symmetric about the midline plane of the well if the midline is the interface between the re-injection and pumping sections. So analysis below will focus only on the pumping section of the well, the analogical conclusions can easily be drawn for the re-injection section.

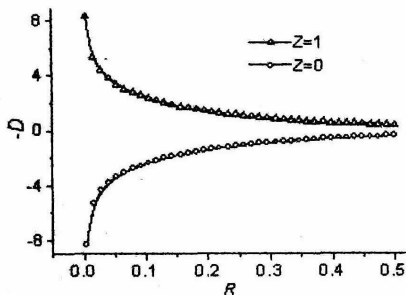
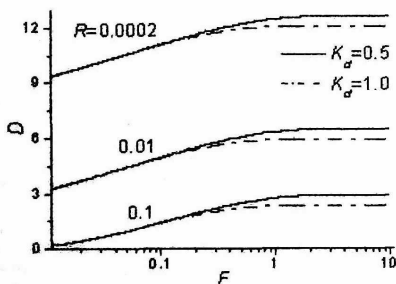
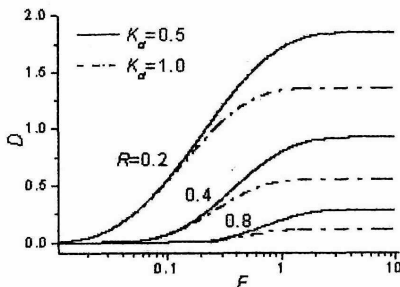


Fig. 4: Dimensionless drawdown profile along the r -direction



a: $Z=0.1, R=0.0002-0.1$



b: $Z=0.1, R=0.2-0.8$

Fig. 5: The dimensionless drawdown versus dimensionless time F and radius R

4. THE STEADY-STATE SOLUTION

When the time approaches infinity, Equation (17) reduces to the corresponding solution for the steady state as follows.

$$S(r, z) = \frac{4Q}{\pi^2 K_r} \sum_{n=1}^{\infty} \left[\frac{\sin(n\pi/2)}{n} \times \cos\left(\frac{n\pi}{l} z\right) K_0\left(\sqrt{\frac{K_z}{K_r}} \frac{n\pi r}{l}\right) \right] \quad (19)$$

where $K_0(x)$ is the modified Bessel function of the second kind of zero order.

The calculated trends of dimensionless drawdown versus dimensionless time and radius are shown in Figure 5. Figure 5-a and Figure 5-b correspond to dimensionless radius $R=0.0002-0.1$ and $R=0.2-0.8$ respectively. A review of these plots reveals that the peak drawdown with K_d of 1.0 is smaller compared to K_d of 0.5, and that the drawdown approaches a steady state after a certain period of time.

Figure 6 shows the relation between the dimensionless variables D and $1/u$. A review of Figure 6 illustrates that increase in the radial distance r , or decrease in $1/u$, results in a smaller drawdown if time τ keeps unchanged. Accordingly, if $R \rightarrow \infty$, then $D \rightarrow 0$; the result conforms to the r-direction boundary condition. If time $\tau = 0$, then $S = 0$, the conclusion conforms to the initial condition.

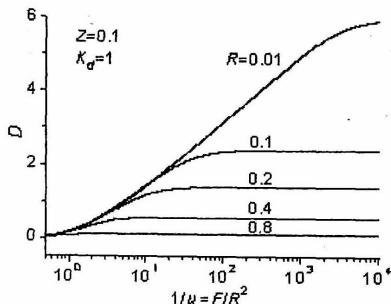


Fig. 6: The relation between the dimensionless drawdown and $1/u$

5. CONCLUSION

The standing column well is a promoting GSHP technology with high efficiency and environmental benefit, where hydrological and geological conditions are suitable. Because of complexity of the seepage flow and heat transfer by groundwater flow in and around the standing column well, this problem has not been analyzed in detail yet.

Determining the seepage flow around standing column wells is a precondition for heat transfer analysis, because the energy equation describing the heat transfer of the system involves the convection term. The seepage flow is an axially symmetrical problem without considering gross groundwater movement. The present paper has presented the analytical solution about this seepage problem by the integral transformation method, which may provide a foundation for heat transfer analysis of the standing column well system.

The results of computation show that the seepage flow appears to reach a steady state after a certain period of operation of the SCW system. The time to reach steady state is decided mainly by properties of the aquifer and r-direction distance. As the radial distance increases, the time to reach the steady state increases.

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