

Analyses on Temperature Distribution around an Inclined Borehole in Ground Heat Exchangers

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Abstract: An analytical solution of the steady-state temperature distribution has been derived in a semi-infinite medium with an inclined line-source of finite length. The impacts of geometrical and physical parameters on this heat transfer process have been discussed accordingly. Two characteristic steady-state temperatures of the inclined borehole wall are defined as the middle section representative temperature and the whole-length integral average temperature. Differences between them are evaluated, and concise expressions for both of them are presented for practical engineering applications.

Keywords: ground-coupled heat pump, ground heat exchanger, heat conduction, inclined boreholes

1. INTRODUCTION

Ground-coupled heat pump (GCHP) systems for building HVAC have aroused more and more interest in recent years due to concern over increasing CO₂ emission caused by fossil-fuel-fired boilers. The efficiency of GCHP systems is inherently higher than that of air source heat pumps because the ground temperature is higher than the average air temperature in winter, when heating is required, and lower in summer, when cooling is required. These systems consist of a sealed loop of pipe, often referred as the ground heat exchanger (GHE), buried in the ground and connected to a heat pump through which water/antifreeze is circulated ^[1]. Consequently, GCHP systems require a certain plot of land for installing the ground heat exchangers. For the GHEs, vertical borehole configuration is usually preferred over horizontal trench systems since it requires less ground areas. In densely populated cities and towns, however, it often remains a serious restriction for application of the GCHP technology to find an adequate plot of land for the GHE. In such situations drilling inclined boreholes may provide a favorable alternative to increase the actual space between adjacent boreholes so as to alleviate the thermal interference among them. While a Swedish researcher carried out a qualitative study on the thermal resistance of inclined boreholes in GHEs ^[2] with some numerical approach, few reports on this subject can be found in literature due to the complication in the heat transfer analysis of the system, which is three-dimensional by nature. The numerical solution, however, is inconvenient for engineering design and simulation purposes.

In vertical and inclined borehole systems the GHE consists of a number of boreholes, each containing a U-tube pipe. The borehole annulus should be grouted so as to provide thermal contact between the pipe

and the soil/rock and to protect groundwater from possible contamination.

In order to figure out thermal performance of the GHEs the most important and fundamental task is to grasp the heat transfer process of a single borehole. Heat transfer in GHEs with multiple boreholes can then be analyzed by means of the superimposition principle. For the sake of difficulty of this problem and its long time scale, which can typically last from several months to several years, the heat conduction process is normally studied in two separated spatial sectors in the models to analyze this process in engineering practice ^[3,4]. One sector is the solid soil/rock outside the borehole. In consideration of the fact that the borehole depth of U-tube is much larger than its diameter, the heat conduction process outside the borehole may be investigated on the assumption that the borehole together with the U-tube in it is simplified as a line heat source or sink. Models of finite ^[2,5] and infinite ^[1] line heat source have been proposed for vertical boreholes. The other sector is that inside the borehole, including the backfilling, the U-tube pipes and the circulating fluid inside the U-tube ^[4]. The analyses on the two spatial sections are interlinked on the borehole wall. Thus, the crucial task for the thermal analysis outside the borehole is to determine the mean temperature rise on the borehole wall under certain operating conditions.

As a simpler but fundamental problem, this study tries to solve analytically the steady-state temperature distribution around an inclined line source with finite length, which is also of practical significance beyond its application in GCHPs like in the case of the vertical one referred as the "buried rod" problem ^[6].

2. THE STEADY-STATE TEMPERATURE DISTRIBUTION AROUND AN INCLINED LINE SOURCE

In a similar way to the vertical borehole analysis^[4,5], the steady-state temperature distribution of an inclined finite line-source model can be solved on the basis of virtual heat source method and linear superposition principle. Assumptions for the model are taken as follows:

- 1) The ground is regarded as a homogeneous semi-infinite medium;
- 2) The boundary of the medium, i.e. the ground surface, keeps a constant temperature;
- 3) The thermophysical properties of the ground do not change with temperature;
- 4) The radial dimension of the borehole is neglected so that it may be approximated as a line-source stretching from the boundary to a certain depth.

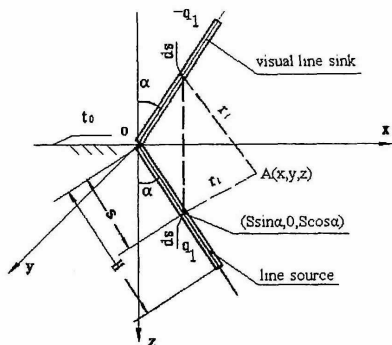


Fig. 1 The geometry of a finite line source in a semi-infinite medium

A diagram of the theoretical model for a single inclined line source is illustrated in Figure 1. One end of the finite line-source is located on the surface of the semi-infinite medium, and is situated at the origin of the coordinate; the line-source lies in the plane xoz, and has an angle of α with the z-axis; its length is H , and the heating rate per length of the source is q_l . The boundary of the medium, i.e. the ground surface, keeps a constant temperature t_0 , so the temperature at infinity should also maintain as t_0 .

Select a differential increment, ds , at the line-source, which can be regarded as a point heat source. The distance between this point and the origin of the coordinate is s as indicated in Figure 1. If the temperature excess is defined as $\theta = t - t_0$, the steady-state temperature excess generated by this point source at a random point $A(x, y, z)$ can be written as^[5,6]

$$d\theta_1 = \frac{q_l ds}{4\pi\lambda r_1} \quad (1)$$

where $r_1 = \sqrt{(x - s \sin \alpha)^2 + y^2 + (z - s \cos \alpha)^2}$.

Based on the virtual heat source principle, a virtual heating rate $-q_l$ on symmetry to the boundary is set as shown in Figure 1. Similarly, the steady-state

temperature excess generated by this line-sink at the same point $A(x, y, z)$ can be written as

$$d\theta_2 = \frac{-q_l ds}{4\pi\lambda r_2} \quad (2)$$

where $r_2 = \sqrt{(x + s \sin \alpha)^2 + y^2 + (z + s \cos \alpha)^2}$.

Because of the linearity of the problem the steady-state temperature excess at the point $A(x, y, z)$ can be regarded as the superposition of the temperature excesses generated by both the line-source and the line-sink according to the linear superposition principle, that is:

$$\theta = \frac{q_l}{4\pi\lambda} \int_0^H \left[\frac{1}{r_1} - \frac{1}{r_2} \right] ds \quad (3)$$

The state-steady temperature field of the inclined finite line-source in a semi-infinite medium can be obtained by substituting the expressions of r_1, r_2 into the Equation (3) and integrating. That yields

$$\theta = \frac{q_l}{4\pi\lambda} \ln \left(\frac{H - x \sin \alpha - z \cos \alpha + r_-}{H - x \sin \alpha + z \cos \alpha + r_+} \cdot \frac{z \cos \alpha - x \sin \alpha + r_-}{-z \cos \alpha - x \sin \alpha + r_+} \right) \quad (4)$$

Where

$$\begin{aligned} r_- &= \sqrt{(x - H \sin \alpha)^2 + y^2 + (z - H \cos \alpha)^2}, \\ r_+ &= \sqrt{(x + H \sin \alpha)^2 + y^2 + (z + H \cos \alpha)^2}, \\ r &= \sqrt{x^2 + y^2 + z^2} \end{aligned}$$

As can be seen in Equation (4), the temperature distribution is a function of the coordinate variables x, y, z and the obliquity α . Besides, it depends also on the heating rate q_l , borehole length H and the thermal conductivity of the medium λ . Furthermore, when $z = 0, \theta = 0$, i.e. the temperature distribution accords with the boundary condition, that the temperature of the ground surface is constant.

Introduce dimensionless variables $X = x/H, Y = y/H, Z = z/H$ and $\Theta = 4\pi\lambda\theta/q_l$, then, Equation (4) may be expressed in a dimensionless form as follows:

$$\Theta = \ln \left(\frac{1 - X \sin \alpha - Z \cos \alpha + R_-}{1 - X \sin \alpha + Z \cos \alpha + R_+} \cdot \frac{-X \sin \alpha + Z \cos \alpha + R_-}{-X \sin \alpha - Z \cos \alpha + R_+} \right) \quad (5)$$

where $R_- = r_-/H = \sqrt{(X - \sin \alpha)^2 + Y^2 + (Z - \cos \alpha)^2}$,

$$R_+ = r_+/H = \sqrt{(X + \sin \alpha)^2 + Y^2 + (Z + \cos \alpha)^2},$$

$$R = r/H = \sqrt{X^2 + Y^2 + Z^2}.$$

As shown in Equation (5), the dimensionless temperature distribution is a function of the dimensionless coordinates X, Y, Z and the obliquity α .

Calculated according to Equation (5), the isotherms around a single inclined line source are plotted in Figure 2, which show clearly the three-dimensional feature of the temperature field. As shown in Figure 2, the closer to the line-source, the denser the isotherms are, which means that the temperature gradients in the vicinity of the line-source are larger. The temperature excess approaches zero at infinity, i.e. equal to the temperature of the ground surface.

When $\alpha = 0$, Equation (5) reduces to:

$$\Theta = \ln \left(\frac{1-Z+\sqrt{X^2+Y^2+(Z-1)^2}}{1+Z+\sqrt{X^2+Y^2+(Z+1)^2}} \cdot \frac{Z+\sqrt{X^2+Y^2+Z^2}}{-Z+\sqrt{X^2+Y^2+Z^2}} \right) \quad (6)$$

which is identical to the solution obtained for the vertical finite line-source [5].

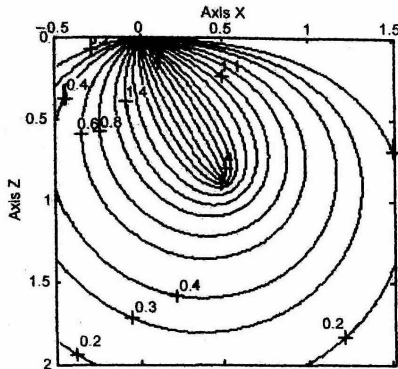


Fig. 2 The steady-state isotherms around an inclined line source

3. CHARACTERISTIC STEADY-STATE TEMPERATURES OF THE BOREHOLE WALL

3.1 The average temperature on the middle section circle of the borehole wall

As can be seen in the derived solutions of the temperature distributions of both the vertical and inclined boreholes, the temperature on the borehole wall varies in the axial direction. It is often required to have a unique representative temperature of the borehole wall for the convenience of engineering calculations. In the heat transfer analysis of the vertical boreholes the temperature in the midpoint of the borehole wall is usually taken as its representative temperature [5, 6]. Different from the temperature distribution of the vertical line source, which is centrally symmetric about its axis, that of the inclined line source is three-dimensional, and then, the temperature is not identical at a borehole wall circle on the cross-section perpendicular to its axis. The dimensionless coordinates of a point on the inclined circle on the middle cross-section may be expressed as:

$$\begin{cases} X = \frac{1}{2} \sin \alpha + R_b \cos \alpha \cos \omega \\ Y = R_b \sin \omega \\ Z = \frac{1}{2} \cos \alpha - R_b \sin \alpha \cos \omega \end{cases}$$

Here ω denotes the angle between the radius passing the concerned point on the circle and the specific radius at the plane xoz , ($0 \leq \omega < 2\pi$). $R_b = r_b/H$ is the

relative radius of the borehole. A mean temperature on this circle can be defined as:

$$\bar{\Theta}_m = \frac{1}{2\pi} \int_0^{2\pi} \Theta(R_b, \alpha, \omega) d\omega \quad (7)$$

This mean temperature is a little complicated to calculate for engineering applications. Therefore, a temperature at the specific point, $M\left(\frac{H}{2} \sin \alpha, r_b, \frac{H}{2} \cos \alpha\right)$, or the point on the circle with $\omega = \pi/2$, is recommended to take the place of its mean temperature. Thus, the middle section representative temperature may be determined simply as:

$$\Theta_m = \ln \left(\frac{\sqrt{1+4R_b^2} + \cos 2\alpha}{\sqrt{1+4R_b^2} + 8\cos^2 \alpha + \cos 2\alpha + 2} \cdot \frac{\sqrt{1+4R_b^2} + 1}{\sqrt{1+4R_b^2} - 1} \right) \quad (8)$$

Figure 3 indicates the relative error between its representative temperature and the mean temperature on the middle section circle as the borehole relative radius and obliquity change. It can be seen from the figure that the relative error between the two proposed temperatures are all less than 7×10^{-6} for a wide range of the obliquity of $\alpha < 60^\circ$ as the relative radius ranges from 0.0005 to 0.005, which is commonly encountered in engineering practice of GHEs. It is clear that the representative temperature, Equation (8), can be used adequately to replace the mean temperature of the middle section circle on the borehole wall, which will facilitate the calculations significantly.

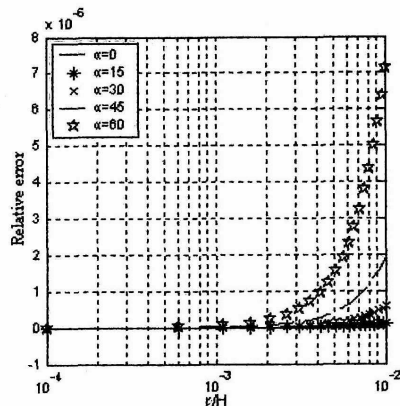


Fig. 3 The relative error between the representative temperature and the integral average temperature of the middle section circle on the borehole wall

3.2 Approximate expressions for the representative temperature of the inclined borehole wall

In the ground heat exchangers the borehole radius r_b is much smaller compared with the

borehole depth H , that is to say, $r_b/H \ll 1$. In this case Equation (8) can be simplified, and the representative temperature of the borehole wall can be simplified as:

$$\Theta_m \approx \ln \left[\frac{1 + \cos 2\alpha}{(\sqrt{1 + 8 \cos^2 \alpha + \cos 2\alpha + 2R_b})} \right], \quad (R_b \ll 1) \quad (9)$$

When $\alpha = 0$, Equation (9) reduces to

$$\Theta_m = \frac{q_l}{4\pi\lambda} \ln \left(\frac{H^2}{3r_b^2} \right), \quad (r_b/H \ll 1) \quad (10)$$

which is the same as the simplified expression for the vertical boreholes [4].

3.3 The integral average temperature over whole length of the borehole

Since the temperature along the length of the borehole wall varies significantly, it is not quite justified to use the temperature at the middle section of the borehole wall to represent its temperature when calculating heat transfer in the whole system. Consequently, the whole-length integral average temperature is recommended here as a more reasonable characteristic temperature of the inclined borehole wall in heat transfer analysis of GHEs. It is defined as

$$\bar{\Theta}_b = \frac{1}{2\pi H} \int_0^H \int_0^{2\pi} \theta(r_b, \omega, s, \alpha) d\omega ds \quad (11)$$

and its dimensionless form $\bar{\Theta}_b = 4\pi\lambda\bar{\Theta}_b/q_l$

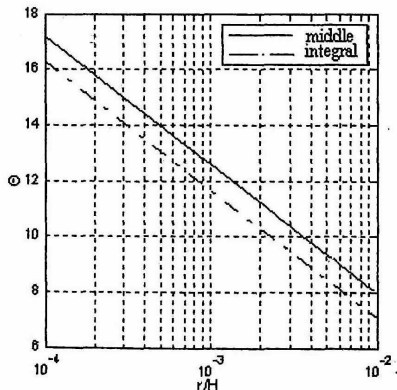


Figure 4: The middle section representative temperature and the whole-length integral average temperature

The relative discrepancy between the representative temperature and the whole-length integral average temperature is defined as:

$$\varepsilon = \frac{\Theta_m - \bar{\Theta}_b}{\bar{\Theta}_b} \times 100\% \quad (12)$$

As can be seen in the Equation (9) and (11), both the dimensionless representative temperature and the whole-length integral average temperature of the inclined borehole are functions of its relative radius, $R_b = r_b/H$, and the obliquity α . The representative temperature of the borehole wall and its whole-length integral temperature are plotted in Figure 4 with $\alpha = 30^\circ$. It shows that the whole-length integral average temperature is a little smaller than the representative temperature. Figure 5 shows the relative discrepancy between the two characteristic temperatures as a function of the obliquity and the relative radius of the inclined borehole. It can be seen that the relative radius, $R_b = r_b/H$, exerts a more decisive impact on the discrepancy than the obliquity as it is usually minor in GHE applications. The relative discrepancy between the two characteristic temperatures ranges from 6.38 percent to 10 percent, as $R_b \leq 0.005$ and $\alpha \leq 30^\circ$ which is likely encountered in engineering practice of GHEs. For a larger R_b , however, the representative temperature at the middle section circle may lead to a substantially greater deviation from the integral average temperature of the borehole.

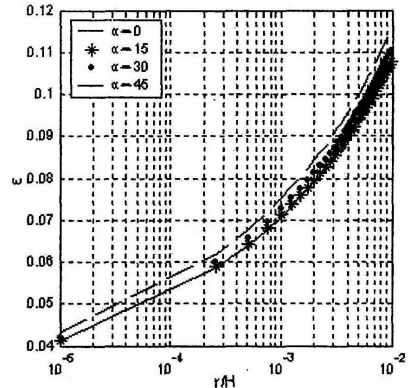


Figure 5: The relative discrepancy of the temperatures versus the relative borehole radius

Although it is defined more reasonably, the whole-length integral average temperature is difficult to calculate, and inconvenient to engineering applications. It can also be seen in Figure 4 that the integral average temperature has an almost linear relation to logarithm of the relative radius of the borehole. Thus, a much simpler semi-empirical expression is derived by means of linear regression of the data computed with Equation (11). The approximate expression takes the following form

$$\bar{\Theta}_b(\alpha) = \frac{q_l}{4\pi\lambda} \ln \left[\frac{\cos \alpha \left(\frac{H}{r_b} \right)^2}{7.07} \right] \quad (r_b/H \ll 1, \alpha \leq \pi/4) \quad (13)$$

The relative deviation of Equation (13) is less than 0.67 percent from the precise values of the whole-length integral average temperature computed with Equation (11) within the range $R_b \leq 0.005$ and $\alpha \leq 45^\circ$. Obviously, the explicit expression of Equation (13) is much more convenient, and even more adequate for engineering applications.

4. CONCLUSION

The ground heat exchanger with inclined boreholes is proposed as a variation of the traditional vertical borehole configuration in the ground-coupled heat pump systems for the purpose of reducing land areas for its installation. The model of line heat source of finite length, which has been used successful for vertical boreholes in GHEs [5], is employed again for the inclined borehole. The steady-state temperature distribution around an inclined line heat source in a semi-infinite medium is analyzed in view of its application in such configurations of the GHEs, especially for their long-term operation. The analytical solution of the temperature distribution of the three-dimensional heat conduction is obtained by means of the Green's function method and superimposition principle. The explicit analytical solution has provided a useful tool for analysis of the heat transfer in the GHEs with inclined boreholes while being also of common significance for similar engineering problems.

Two characteristic temperatures are considered to represent the temperature rise on the borehole wall, i.e. the representative temperature at a specific point on the middle section circle of the borehole wall and the integral average temperature over its whole length. Both the steady-state temperatures can be used for analysis of the long-term performance of the GHEs. A possible difference up to 10 percent may exist between these two characteristic temperature excesses in the common GHE applications. The representative temperature at the middle of the borehole wall of Equation (9) is simpler in definition

and accords with the traditional approach to the vertical borehole, or "buried rod" [6] problems. The integral average temperature of the borehole wall is more properly defined for heat transfer analysis although it is inconvenient for direct calculations. A semi-empirical function in an explicit and simple expression with satisfactory accuracy, however, is presented, describing the steady-state integral average temperature of the inclined borehole wall. Therefore, this expression is highly recommended for engineering applications.

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