

The Research of the Transfer Performance of the Heat Exchanger

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Abstract: There are three kinds of methods to study the transfer performance of the heat exchanger. They are Equal Reynolds Method, Wilson Method and Least-square Method. They have their own working condition and principle. According to some test data, which is collected in one plate heat exchanger, this paper compares the results of the three methods. At last a conclusion is draw—the three methods are associated and using the three methods comprehensive consideration should be given.

Keywords: transfer performance; equal Reynolds method; Wilson method; least-square method; heat exchanger

1. INTRODUCTION

We often use experimental methods to obtain the equations of the heat transfer by convection. However, in order to get the equations of function of different variables, we must do so many experiments that they cannot realize in fact. According to the theoretical analyses of the laws of similitude and the internal relation of variables, we can reduce the number of the variables. But there is an essential premise—we should know the rule of the phenomena. In the light of the laws of similitude, the transfer performance of heat exchanger can be expressed by the exponential function:

$$Nu = C Re^m Pr^n \quad (1)$$

where: Nu —the Nusselt number;
 Re —the Reynolds number;
 Pr —the Prandtl number;
 C, m, n —the constant.

How to get the function is very important. Now three useful methods are introduced.

2. EQUAL REYNOLDS METHOD

2.1 Working condition

This method requires two fluid passages of heat exchanger should be geometrical similitude. If two passages of a heat exchanger have the same structure, when velocities of two fluids are equal, the Reynolds numbers of the hot and cold fluids (water) are equal. In this time the equal Reynolds method can be used to get the coefficient of the heat transfer by convection.

2.2 Principle of the method

The equation (1) can be used to describe the state of convection in the heat exchanger that has the geometrical similitude passage. In this function $n=0.4$ when the fluid is hot and $n=0.3$ when the fluid is cold.

The subscript 1 and 2 denote hot fluid and cold fluid respectively, then the formula (1) can be changed into

formula (2) and (3).

$$h_1 = C Re_1^{m_1} Pr_1^{n_1} \frac{\lambda_1}{d_1} \quad (2)$$

$$h_2 = C Re_2^{m_2} Pr_2^{n_2} \frac{\lambda_2}{d_2} \quad (3)$$

where: h —the coefficient of heat transfer by convection, $W/(m^2 \cdot K)$;

λ —the coefficient of heat transfer by conduction, $W/(m \cdot K)$;

d —the equivalent diameter, m.

The thermal resistance equation of heat transfer is:

$$\frac{1}{k} = \frac{1}{h_1} + \frac{\delta}{\lambda} + \frac{1}{h_2} \quad (4)$$

where: k —the heat transfer coefficient, $W/(m^2 \cdot K)$;

δ —the thickness, m.

If the Reynolds numbers are equal,

$Re_1 = Re_2 = Re$. Connect the equation (2) and (3) with equation (4), the equation (2) can be changed into the equation (5):

$$C Re^n = \frac{1}{(1/k) - (\delta/\lambda_w)} \left[\frac{1}{(\lambda_1/d_1) Pr_1^{0.3}} + \frac{1}{(\lambda_2/d_2) Pr_2^{0.4}} \right] \quad (5)$$

Use the variable P ,

$$P = \frac{1}{(1/k) - (\delta/\lambda_w)} \left[\frac{1}{(\lambda_1/d_1) Pr_1^{0.3}} + \frac{1}{(\lambda_2/d_2) Pr_2^{0.4}} \right]$$

Calculate the logarithm of every term of equation (5), then

$$\ln P = \ln C + m \ln(Re) \quad (6)$$

The formula (6) is liner function. Through some experiments, the data $(\ln P_i, \ln Re_i)$ ($i=1,2,\dots$) can be get. The constant C and m can be obtained by the linear regression of equation (6).

3 WILSON METHOD

3.1 Working condition

- (1) the coefficient of the convection heat transfer, constant C and m (in the equation (2)) of one fluid must be known.
- (2) In the same group experiments, the experimental condition of the other fluid cannot be changed.
- (3) In the same group experiments, the scale thermal resistance cannot be changed.

3.2 Principle of the method

The equation (4) can be used to calculate the coefficient of the heat transfer. According to the result of Reynolds method, the equation (7) can be obtained:

$$h_2 = c_2 f(u_2) \quad (7)$$

Then the function (4) can be changed into

$$\frac{1}{k} = \frac{1}{\alpha_1} + \frac{\delta}{\lambda} + \frac{1}{c_2 f(u_2)} \quad (8)$$

Keep the heat transfer coefficient of one side be constant, and change the velocity of the fluid in the other side, then the formula (8) can be changed into

$$\frac{1}{k} = b + \frac{1}{c_2 f(u_2)} \quad (9)$$

where: $b = 1/\alpha_1 + \delta/\lambda$ (it is constant).

The formula (9) is the liner equation. It also can be denoted by the $Y = b + MX$, here $Y = 1/k$, $X = 1/f(u_2)$.

The different experimental results are drawn in a figure, the liner equation can be obtained from the figure (using the computer knowledge). The scope of the liner equation (M) can be known. The other constant c_2 can be get by the formula (10)

$$c_2 = 1/M \quad (10)$$

Now the heat transfer coefficient of the other fluid can be calculated in the equation (7).

Next get the result of the logarithm of the equation (4),

$$\ln Nu = \ln C + m \ln Re + n \ln Pr \quad (11)$$

here $n=0.4$ when the fluid is hot and $n=0.3$ when the fluid is cold.

In the equation (11) the $Nu = \alpha d/\lambda$,

$Re = ud/\nu$ and Pr can be calculated in terms of the experimental data, so the equation (11) can be changed into

$$\ln Nu - n \ln Pr = \ln C + m \ln Re \quad (12)$$

Make it be liner, then

$$Y = c + mX \quad (13)$$

Here $Y = \ln Nu - n \ln Pr$; $X = \ln Re$; $c = \ln C$.

In the same method, the constant m and C of the equation (13) can be known by computer calculation.

4 LEAST SQUARE METHOD

4.1 Prepare of the data

Firstly calculate the quantity of the heat exchange, the formulae are:

$$\Phi_1 = c_{p1} \cdot \rho_1 \cdot G_1 \cdot (t_1' - t_1'') \quad (14)$$

$$\Phi_2 = c_{p2} \cdot \rho_2 \cdot G_2 \cdot (t_2'' - t_2') \quad (15)$$

where: t_1' , t_1'' — the temperature of the entry and exit of the heat exchanger, °C;

c_p — the specific heat;

ρ — density;

G — flow rate of volume.

Because of some reason the heat absorption of the cold fluid does not equalize the output of heat of the hot fluid, the arithmetic mean value ($\Phi = (\Phi_1 + \Phi_2)/2$) can be used as the capacity of heat transmission, then the density q can be calculated by the following formula:

$$q = \Phi / F \quad (16)$$

4.2 Pilot calculation of the temperature of wall

Suppose the temperature of wall as t_{w1} , the Newton cooling formula is

$$q = h_1(t_{w1} - t_1') \quad (17)$$

where: t_w — the temperature of wall, °C;

t — the mean temperature of the entry and exit of the fluid, °C.

Now the convection heat transfer coefficient in the hot fluid can be calculate, that is h_1 . According to the Fouriers Law,

$$q = \frac{(t_{w1} - t_{w2}) \cdot \delta}{\lambda} \quad (18)$$

the temperature of wall in the cold fluid (t_{w2}) can be calculated. Use the same method as that of calculating h_1 to get h_2 . Then use the equation of the thermal resistance of heat transfer (equation (19))

$$\frac{1}{kk} = \frac{1}{\alpha_1} + \frac{\delta}{\lambda} + \frac{1}{\alpha_2} \quad (19)$$

to calculate the heat transfer coefficient kk . Compare k with kk until they are equal (the equal means the error is very little). Now use h_1 and h_2 to continue calculating.

4.3 Least square function

Change the formula (11) into

$$z' = d + mx + ny \quad (20)$$

where: $z' = \ln Nu$, $x = \ln Re$, $y = \ln Pr$.

If there are k groups of data (Nu_1, Re_1, Pr_1) , (Nu_2, Re_2, Pr_2) , ..., (Nu_k, Re_k, Pr_k) ,

respectively calculate their logarithm value and change them into $(z_1, x_1, y_1), (z_2, x_2, y_2), \dots, (z_k, x_k, y_k)$. Use i describe the number of the experiments, then use $z_1, z_2, \dots, z_k$ as the value of $\lg Nu$.

Then

$$W = \sum_{i=1}^k (z_i - z_i')^2 = \sum_{i=1}^k (d + mx_i + ny_i - z_i')^2 \quad (21)$$

Here W denotes the sum of squares of difference between the data of experiment and calculation, in order to get the minimum of the W , respectively calculate the following value of the derivative.

$$\frac{\partial W}{\partial m} = 0 \quad \frac{\partial W}{\partial n} = 0 \quad \frac{\partial W}{\partial d} = 0$$

And $n=0.4$ when the fluid is hot and $n=0.3$ when the fluid is cold. So next three formulae can be get

$$m \sum_{i=1}^k x_i^2 + d \sum_{i=1}^k x_i = \sum_{i=1}^k z_i x_i - n \sum_{i=1}^k x_i y_i \quad (22)$$

$$m \sum_{i=1}^k x_i y_i + d \sum_{i=1}^k y_i = \sum_{i=1}^k z_i y_i - n \sum_{i=1}^k y_i^2 \quad (23)$$

$$m \sum_{i=1}^k x_i + dk = \sum_{i=1}^k z_i - n \sum_{i=1}^k y_i \quad (24)$$

Work out the equations, then m and n can be obtained.

5. COMPARE OF THE THREE METHODS

Do experiment in the BR0.05 type of plate heat exchanger and use the Equal Reynolds Method, Wilson Method and Least-square Method to calculate the equation of heat transfer and the equation of fluid resistance:

(1) the Equal Reynolds Method

$$Nu = 0.2157 Re^{0.5769} Pr^n \quad (25)$$

where: $Re=5 \times 10^3 \sim 5 \times 10^4$

(2) the Wilson Method

$$Nu = 0.2329 Re^{0.5832} Pr^n \quad (26)$$

where: $Re=5 \times 10^3 \sim 2.5 \times 10^4$

(3) the Least-square Method

$$Nu = 0.2327 Re^{0.5772} Pr^n \quad (27)$$

where: $Re=5 \times 10^3 \sim 5 \times 10^4$

(notes: $n=0.4$ when the fluid is hot and $n=0.3$ when the fluid is cold.)

Compare three equations (25),(26),(27) in the figure 1

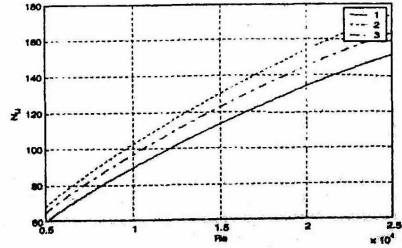


Figure 1 the compare of three methods (1-the Equal Reynolds Method, 2- the Wilson Method, 3- the Least-square Method)

From the statistical point of view, compare the residual error of original data with that of the calculated data, figure 2 and 3 can be drawn.

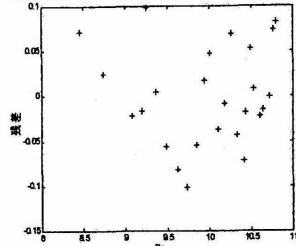


Figure 2 the residual error of the Equal Reynolds Method

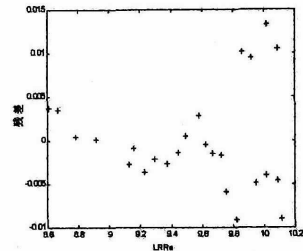


Figure 3 the residual error of the Wilson Method

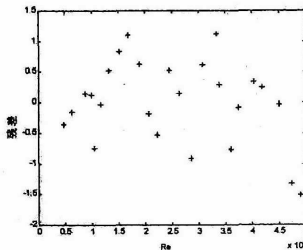


Figure 4 the residual error of the Least-square Method

According to the experimental data, the variance can be calculated.

s1=49.845	(the Equal Reynolds Method)
s2=19.72	(the Wilson Method)
s3=26.63	(the Least-square Method)

6.CONCLUSION

From the figure 1 the result of the Equal Reynolds Method is less 16% ~ 20% than that of the Least-square Method and less 25%~33% than that of the Wilson Method. From the figure 2,3,4 and the value of the variance, the residual error and the variance of the Wilson Method is lest. So the regression accuracy of the Wilson Method is best. But looking back the

calculating process of the Wilson Method, we can find that the Wilson Method is connected with the Equal Reynolds Method. Therefore, we should integrate different methods so that we can get the best result.

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