

Forecasting Energy Consumption of China Based on Grey System Theory

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ABSTRACT: Based on the actual energy consumption of China from 1998 to 2006, the GM (1,1) forecasting models for the total energy, coal energy and clean energy consumption of China are established successfully, with an precision check-up thereafter. The result shows the three forecasting models are all in first precision level, which is suitable for simulating and forecasting the original data sequences of the energy consumption characterized by grey type. It is known from the result that the energy consumption of China is at the exponential growth rate. From 2007 to 2012, the total energy, coal energy and clean energy consumption of China will increase by 13.19%, 13.78% and 14.29% respectively; and in the energy consumption structure, the proportion of the coal energy and clean energy consumption will increase by 0.33% and 0.65% respectively. The GM (1,1) models predict this trend and show the proportion of the clean energy in the total energy consumption and its increment speed is escalating in China. This conforms to the future sustainable multiple clean energy consumption strategy of the world based mainly on coal, also provides a reference for China to solve the problems of energy supply shortage and environmental pollution.

Keywords: GM (1,1) model; clean energy; energy consumption; prediction

1. INTRODUCTION

Energy is the most important national strategic resource as well as the key material basis for increasing the state economy and social development. For nearly one decade, the economy of China has been developing at an incredible average rate of 10% ^[1], which rapidly increases the total energy consumption. As one of the biggest developing country, China holds the third position in the energy production and the second position in the basic energy

consumption in the world (11% of the world total consumption). In this case, China has already been a great power for energy production and consumption. But for a long time, the energy consumption of China mainly depends on coal, such unsuitable energy production and consumption structure finally results in severe environmental pollution, also partly restricts the general economic development of China. Therefore, objectively and correctly forecasting the energy consumption volume definitely has important theoretical and

practical significance for China to properly layout the energy consumption structure, coordinate the relationship between the energy consumption and economic development, and establish countermeasures to fight against the energy related pollution. Meanwhile, the growth trend of China's energy consumption will influence the future global economic development and energy consumption; also provide a reference for other countries to establish the corresponding energy production and consumption strategies.

The grey system is defined as the uncertain system of "partial information known, partial information unknown". The grey forecasting model takes the grey system as the study subject, and according to the real data and the "partial known" information to exploit, form and extract the valuable information, so as to realize the correct description of the system operation behaviour and evolution order^{[2]-[4]}. Currently the grey system theory has multiple applications in numerous areas, also achieves many favourable fruits^{[5]-[7]}. The energy consumption, as a very complicated system, is featured by its information uncertainty and few useful analyzing samples. Hence it is reasonable to build the consumption forecasting models of clean energy, coal energy and total energy of China according to the energy consumption history data.

2. BASIC PRINCIPLE OF GM (1,1) MODEL

2.1 What is GM (1,1) Model?

GM model is the essential part of the grey system theory. This model takes the grey module as its foundation, organizes the seemingly discrete data sequences into the regular sequences via data processing. The differential equations are built for the new sequences and the calculation values of model are obtained. And by compared with the real measured values, the residual errors

are obtained. Using the residual errors to correct the model, then the model is endowed with a higher forecasting precision level^[4]. There are many forecasting grey models in the grey system, but the most commonly used one is GM (1,1) model, which is also the foundation of other grey forecasting models. GM (1,1) is an "one order one variable" grey system model that is compatible with the grey differentiation and time-varying difference. This model also has some features of differential equations, can preferably describe the interior characters and developing trend of the system, and possesses preferable extrapolating predictability as well.

2.2 Establishment of GM (1,1) Model

1) Generation of the initial sequences. By observing certain equal time interval segments of the system, a set of numerical sequences are obtained, named as original numerical sequences $X^{(0)}$:

$$X^{(0)} = \{x^{(0)}(k)\} (k=1, 2, \dots, n) \quad (1)$$

Let $X^{(0)}$ be a primal time sequence, and obtain its first-order AGO (Accumulated Generating Operation) data $X^{(1)}$:

$$X^{(1)} = \{x^{(1)}(k)\} (k=1, 2, \dots, n) \quad (2)$$

$$\text{Where : } X^{(1)}(k) = \sum_{i=1}^k x^{(0)}(i)$$

The advantage of this operation lies in the reduction of the randomness of the original time sequence. If $X^{(0)}$ is a negative sequence, then $X^{(1)}$ is an increasing sequence.

2) Make quasi-smoothness examination and exponential analyses on $X^{(1)}$, ascertain whether it is possible to establish the GM (1,1) model. According to:

$$\rho(k) = \frac{x^{(0)}(k)}{x^{(1)}(k-1)}, k=2, 3, \dots, n-1 \quad (3)$$

When $\rho(k) \in [0, \varepsilon]$, $k=3, 4, \dots, n$, and $\varepsilon < 0.5$, we name $X^{(1)}$ as the quasi-smoothness sequence. Also according to:

$$\sigma^{(1)}(k) = \frac{x^{(1)}(k)}{x^{(1)}(k-1)}, k=2, 3, \dots, n-1 \quad (4)$$

When $\sigma^{(1)}(k) \in [a, b]$, $k = 3, 4, \dots, n$, and $\delta = b - a = 0.5$, $X^{(1)}$ follows the quasi-exponential order. When $X^{(1)}$ satisfies the quasi-smoothness sequence and quasi-exponential order at same time, the GM (1,1) model can be established.

3) Make the least-squares estimation on the parameter sequence $\hat{a} = [a, b]^T$, obtain:

$$\hat{a} = (B^T B)^{-1} B^T Y \quad (5)$$

Where

$$B = \begin{bmatrix} -Z^{(1)}(2) & -Z^{(1)}(3) & \cdots & -Z^{(1)}(n) \\ 1 & 1 & \cdots & 1 \end{bmatrix}^T;$$

$$Z^{(1)}(k) = 0.5x^{(0)}(k) + 0.5x^{(0)}(k-1);$$

$$Y = [x^{(0)}(2), x^{(0)}(3), \dots, x^{(0)}(n)].$$

$x^{(0)}(k) + az^{(1)}(k) = b$ is called as the differential equation of GM(1,1), it is the essential form of the GM(1,1) grey forecasting model.

4) Determine the whitenization equation and time response formulae of $X^{(1)}$, build the GM (1,1) forecasting model.

$$\begin{cases} \frac{dx^{(1)}}{dt} + ax^{(1)} = b \end{cases} \quad (6)$$

$$\begin{cases} \hat{X}^{(1)}(k) = \left(X^{(0)}(1) - \frac{b}{a} \right) e^{-a(k-1)} + \frac{b}{a} \end{cases} \quad (7)$$

$$\begin{cases} \hat{X}^{(0)}(k) = \hat{X}^{(1)}(k) - \hat{X}^{(1)}(k-1) \end{cases} \quad (8)$$

Where: $-a$ is the development coefficient; b is the grey action quantity; $\hat{X}^{(1)}$ is the simulative value of $X^{(1)}$; while $\hat{X}^{(0)}$ is the simulative value of $X^{(0)}$. Therein make $x^{(1)}(1) = x^{(0)}(1)$, make I-AGO(Inverse Accumulated Generating Operation) on $\hat{X}^{(1)}$, obtain $\hat{X}^{(0)}$. The above equation is the mathematic expression of GM (1,1) model.

2.3 Accuracy check-up for GM (1,1) Model

The established GM (1,1) models must be checked to see if they satisfy the accuracy requirement. We usually check the GM (1,1) model accuracy by the methods such as the “posterior error”, “residual error” and “degree of grey association”.

This article takes the “posterior error” method as the tool to check the model accuracy^[8], the steps are given as below:

1) Calculate the average value $\bar{X}$ and mean square deviation S_1^2 of the original sequence:

$$\bar{X} = \frac{1}{n} \sum_{k=1}^n X^{(0)}(k) \quad (9)$$

$$S_1^2 = \frac{1}{n} \sum_{k=1}^n \left(X^{(0)}(k) - \bar{X} \right)^2 \quad (10)$$

2) Calculate the average value $\bar{\varepsilon}$ and mean square deviation S_2^2 of the residual errors:

$$\bar{\varepsilon} = \frac{1}{n} \sum_{k=1}^n \varepsilon^{(0)}(k) \quad (11)$$

$$S_2^2 = \frac{1}{n} \sum_{k=1}^n \left(\varepsilon^{(0)}(k) - \bar{\varepsilon} \right)^2 \quad (12)$$

3) Calculate the mean square deviation ratio C and small error probability p :

$$C = \frac{S_2}{S_1} = \sqrt{\frac{S_2^2}{S_1^2}} \quad (13)$$

$$p = P\{|\varepsilon(k) - \bar{\varepsilon}| < 0.6745 S_1\} \quad (14)$$

4) Check the models by C, P values.

Small C value usually means the discreteness of the prediction errors is small; while large P value usually means the probability of the smaller errors is big. So, the smaller the C value, the bigger the P value, and the higher the forecasting precision. Table I gives the reference values of the forecasting precision levels. So far as C, P are staying in the allowed scope, the GM (1,1) model can be used to make the prediction.

3. APPLICATIONS OF GM (1,1) ON CHINA ENERGY CONSUMPTION PREDICTION

According to the total energy consumption and structure data in the *Statistical Yearbook 2007 China*, we adopt the real data from 1998 to 2006 to form the

original sequence $X^{(0)}$. Then follow the above steps to build the GM (1,1) models of the clean energy, coal energy, and total energy consumption, and forecast the consumption amount for the future 6 years. Now take the clean energy consumption as an example, build the model and calculate it.

The known clean energy consumption from 1998 to 2006 is:

$$\begin{aligned} X^{(0)} &= (x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(9)) \\ &= (8858, 8298, 9283, 11312, 11688, \\ &\quad 11893, 14429, 15952, 17731) \end{aligned}$$

Make 1-AGO on $X^{(0)}$ and get :

$$\begin{aligned} X^{(1)} &= (x^{(1)}(1), x^{(1)}(2), \dots, x^{(1)}(9)) \\ &= (8858, 17156, 26439, 37751, \\ &\quad 49439, 61332, 75761, 91713, 109444) \end{aligned}$$

According to equation (3), by quasi-smoothness examination on $X^{(1)}$, we know:

$$\begin{aligned} \rho(3) &\approx 0.54; \rho(4) \approx 0.43; \rho(5) \approx 0.31; \\ \rho(6) &\approx 0.24; \rho(7) \approx 0.24; \rho(8) \approx 0.21; \\ \rho(9) &\approx 0.19. \end{aligned}$$

According to equation (4), by exponential order examination on $X^{(1)}$, we know:

$$\begin{aligned} \sigma(3) &\approx 1.54; \sigma(4) \approx 1.43; \sigma(5) \approx 1.31; \\ \sigma(6) &\approx 1.24; \sigma(7) \approx 1.24; \sigma(8) \approx 1.21; \\ \sigma(9) &\approx 1.19. \end{aligned}$$

When $k > 3$, $\rho(k) < 0.5$, $\sigma(k) \in [1, 1.5]$, $\xi = 0.5$. $X^{(1)}$ satisfies the two requirements of quasi-smoothness sequence and quasi-exponential order, so the GM (1,1) can be established.

According to equation (5), calculate generated sequence of immediate neighbouring average value:

$$\begin{aligned} Z^{(1)} &= (z^{(1)}(2), z^{(1)}(3), \dots, z^{(1)}(9)) \\ &= (13007, 21798, 32095, 43595, \\ &\quad 55386, 68547, 83737, 100579) \end{aligned}$$

Hence we can figure out :

$$\begin{aligned} B &= \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(9) & 1 \end{bmatrix} = \begin{bmatrix} -13007 & 1 \\ -21798 & 1 \\ -32095 & 1 \\ -43595 & 1 \\ -55386 & 1 \\ -68547 & 1 \\ -83737 & 1 \\ -100579 & 1 \end{bmatrix} \\ Y &= \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(9) \end{bmatrix} = \begin{bmatrix} 8298 \\ 9283 \\ 11312 \\ 11688 \\ 11893 \\ 14429 \\ 15952 \\ 17731 \end{bmatrix} \end{aligned}$$

Then the parameter sequence can be further determined :

$$\hat{a} = [a, b]^T = (B^T B)^{-1} B^T Y = \begin{bmatrix} -0.104537 \\ 7101.761244 \end{bmatrix}$$

According to equation (7), we obtain the GM (1,1) model:

$$\begin{aligned} \hat{X}^{(1)}(k) &= \left(X^{(0)}(1) - \frac{b}{a} \right) e^{-a(k-1)} + \frac{b}{a} \\ &= 76793.38407 e^{0.104537(k-1)} - 67935.38407 \end{aligned}$$

By further calculation, we obtain:

$$\begin{aligned} \hat{X}^{(1)} &= (x^{(1)}(1), x^{(1)}(2), \dots, x^{(1)}(9),) \\ &= (8858, 17320, 26715, 37146, \\ &\quad 48725, 61581, 75853, 91698, 109289) \end{aligned}$$

According to equation (8), make I-AGO on $\hat{X}^{(1)}$, we obtain the simulative value of $x^{(0)}$:

$$\begin{aligned} \hat{X}^{(0)} &= (x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(9)) \\ &= (8858, 8462, 9395, 10431, 11579, \\ &\quad 12856, 14272, 15845, 17591) \end{aligned}$$

According to equation (9) to (14), examine this model by “posterior error” method, we know:

$$\begin{aligned} \bar{X} &\approx 12161; S_1^2 \approx 9519135.67; \\ \bar{e} &\approx 17.22; S_2^2 \approx 200843.44. \end{aligned}$$

Hence, the mean square deviation ratio is:

$$C = \frac{S_2}{S_1} \approx 0.15 < 0.35;$$

Table I : The grade standard of GM (1,1) model precision test

Accuracy class	Posterior mean square deviation ratio C	Small error probability P
First (Good)	<0.35	>0.95
Second (Qualified)	$0.35 \sim 0.50$	$0.95 \sim 0.80$
Third (scarcely qualified)	$0.50 \sim 0.65$	$0.80 \sim 0.70$
Fourth (unqualified)	>0.65	<0.70

Table II : Clean energy, coal energy and total energy consumption from 1998 to 2006, China.
Unit: (ten thousand tons of standard coal)

Year	Clean energy	Coal energy	Total energy
1998	8858	92021	132214
1999	8298	92477	133831
2000	9283	93939	138553
2001	11312	95514	143199
2002	11688	100642	151797
2003	11893	119693	174990
2004	14429	138195	203227
2005	15952	155255	224682
2006	17731	170912	246270

Small error probability is:

$$P\{|\varepsilon(k) - \bar{\varepsilon}| < 0.6745S_1\} = 1.$$

According to Table I, the accuracy of the established GM (1,1) model is at the first level, which means it can be used to make the prediction. Additionally, according to the forecasting standard of the grey system, when the development coefficient $-a < 0.3$, GM (1,1) model can be used in the mid long term forecasting^[9]. The development coefficient of this model is $-a = 0.104537 <$

0.3, therefore, we can use this model to forecast the clean energy consumption of China from 2007 to 2010. The calculation result is as follows:

$$\begin{aligned}\hat{X}^{(0)} &= (x^{(0)}(10), x^{(0)}(11), \dots, x^{(0)}(15)) \\ &= (19530, 21682, 24071, 26723, 29668, 32938)\end{aligned}$$

Similarly, we can build the GM (1,1) model of coal energy consumption and total energy consumption of China from 2007 to 2010. The results are showed in the Table III, Table IV and Table V.

Table III: Simulative values and errors of GM (1,1) model for China clean energy consumption
Unit: (ten thousand tons)

No.	Year	original quantity $x^{(0)}(k)$	Simulated quantity $\hat{x}^{(0)}(k)$	Absolute error $\varepsilon(k) = x^{(0)}(k) - \hat{x}^{(0)}(k)$	Relative error $\Delta k = \varepsilon(k) / x^{(0)}(k)$
1	1998	8858	8858	0	0
2	1999	8298	8462	-164	1.98%
3	2000	9283	9395	-112	1.21%
4	2001	11312	10431	881	7.79%
5	2002	11688	11579	109	0.93%
6	2003	11893	12856	-963	8.09%
7	2004	14429	14272	157	1.09%
8	2005	15952	15845	107	0.67%
9	2006	17731	17591	140	0.79%

Average error	2.51%
Model accuracy	97.49%

Table IV: Simulative values and errors of GM (1,1) model for China coal energy consumption
Unit: (ten thousand tons of standard coal)

No.	Year	Original quantity $x^{(0)}(k)$	Simulated quantity $\hat{x}^{(0)}(k)$	Absolute error $\varepsilon(k) = x^{(0)}(k) - \hat{x}^{(0)}(k)$	Relative error $\Delta k = \varepsilon(k) / x^{(0)}(k)$
1	1998	92021	92021	0	0
2	1999	92477	81443	11034	11.93%
3	2000	93939	90312	3627	3.86%
4	2001	95514	100147	-4633	4.85%
5	2002	100642	111054	-10412	10.35%
6	2003	119693	123149	-3456	2.89%
7	2004	138195	136559	1636	1.18%
8	2005	155255	151431	3824	2.46%
9	2006	170912	167923	2989	1.75%
Average error					4.36%
Model accuracy					95.64%

Table V: Simulative values and errors of GM (1,1) model for China total energy consumption
Unit: (ten thousand tons of standard coal)

No.	Year	Original quantity $x^{(0)}(k)$	Simulated quantity $\hat{x}^{(0)}(k)$	Absolute error $\varepsilon(k) = x^{(0)}(k) - \hat{x}^{(0)}(k)$	Relative error $\Delta k = \varepsilon(k) / x^{(0)}(k)$
1	1998	132214	132214	0	0
2	1999	133831	121586	12245	9.15%
3	2000	138553	126258	12295	8.87%
4	2001	143199	148248	-5049	3.53%
5	2002	151797	163698	-11901	7.84%
6	2003	174990	180758	-5768	3.29%
7	2004	203227	199596	3631	1.79%
8	2005	224682	220396	4286	1.91%
9	2006	246270	243365	2905	1.18%
Average error					4.17%
Model accuracy					95.83%

Table VI: The predictive values of the three GM (1,1) modes and the proportions of cleaning energy and coal energy consumption
Unit: (ten thousand tons of standard coal)

Year	Total energy	Clean energy		Coal energy	
		Quantity	Prop.	Quantity	Prop.
2007	268727	19530	69.29%	186210	7.27%
2008	296732	21682	69.59%	206489	7.31%
2009	327656	24071	69.88%	228977	7.35%
2010	361802	26723	70.18%	253913	7.39%
2011	399507	29668	70.48%	281565	7.43%
2012	441141	32938	70.78%	312229	7.47%

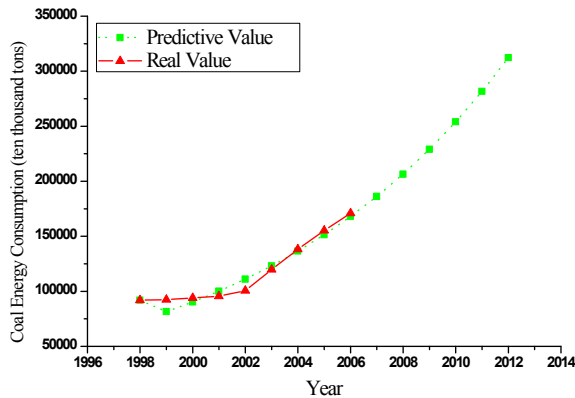


Figure 1: Fitting curve of total energy consumption, 1998-2010, China.

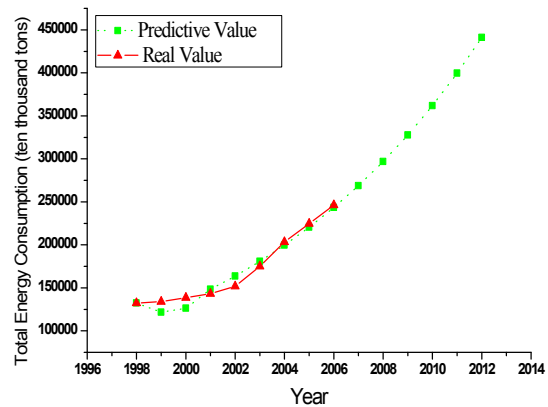


Figure 2: Fitting curve of coal energy consumption, 1998-2010, China

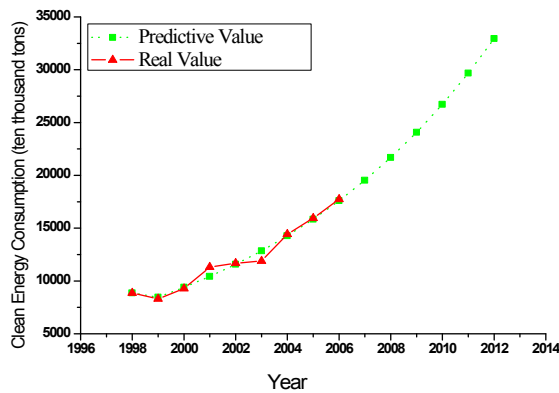


Figure 3: Fitting curve of clean energy consumption, 1998-2010, China.

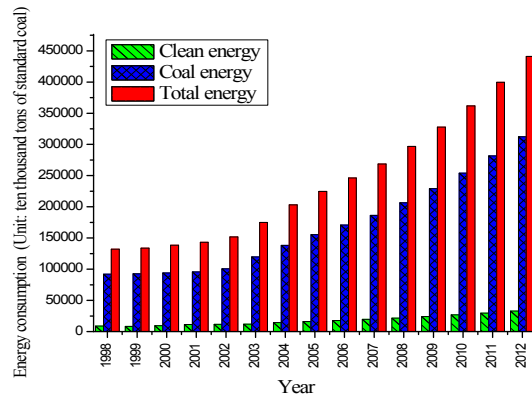


Figure 4: Increasing trend of clean energy, coal energy and total energy consumption, 1998-2012, China

4. CONCLUSION

1) By applying grey system theory, and according to the actual energy consumption quantity of China from 1998 to 2006, the GM (1,1) forecasting models for clean energy, coal energy and total energy consumption of China were respectively built as follows:

$$\hat{X}_1^{(1)}(k) = 76793.38407e^{0.104537(k-1)} - 67935.38407$$

$$\hat{X}_2^{(1)}(k) = 747840.5491e^{0.103372(k-1)} - 655819.5491$$

$$\hat{X}_3^{(1)}(k) = 1166693.837e^{0.099134(k-1)} - 1034479.837$$

2) By using the "posterior error" method to examine established models, and the result showed that the accuracy of the three

forecasting models were all up to the first precision level. The simulation accuracies of the simulative values to the actual values are 97.49%, 95.64% and 95.83% respectively. These models all met the mid long term prediction standard, and forecasting results were relatively good, can reasonably reflect the growth trend of the future energy consumption of China.

3) The clean energy, coal energy and total energy consumption of China from 2007 to 2012 were forecasted according to the established forecasting models. The results show that the total energy, coal energy and clean energy consumption will increase at the rate of 13.19%, 13.78% and 14.29% respectively in the future 6 years. In

the energy consumption structure, the proportion of the coal energy and clean energy consumption would also increase gradually (Table VI), the growth rates are 0.33% and 0.65% respectively. This means that along with the rapid development of China's economy, the need for energy is increasing continuously. Together with the growth of the coal energy consumption, the proportion of the clean energy consumption is also growing rapidly with a growth rate of 0.51% higher than the coal energy. In the total energy consumption, the growth rate of the clean energy is 0.32% higher than the coal energy.

4) The GM (1,1) model used in this article is the most common model in the grey system theory, in which we also had GM(1,N), GM(0,N), GM(2,1) and Verhulst models etc.. Choice of the models should base on the sufficient qualitative analytical conclusions ^[9]. When the accuracy of the GM (1,1) models do not meet the requirements, we must use the residual error sequences to correct the original models, so as to improve the forecasting accuracy.

5) In future, the China's energy consumption structure will still mainly depend on coal, together with the obviously upward trend of the consumption share and increasing pace of the clean energy. This conforms to the future sustainable multiple clean energy consumption strategy that mainly based on coal in the world. This fact also provides a reference for China to solve the problems of energy supply shortage and environmental pollution, as well as sets an example for other countries to establish and adjust the energy consumption structure, to coordinate the relationship among energy, economy and environment.

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